Return by 12.00, Wednesday 6.4.2022 (electronically to pyry.m.rahkila@jyu.fi).

1. **Geodesics in de Sitter:** Consider a test particle on a geodesic path $\chi = \theta = \pi/2$, $\phi = 3\pi/2$ in dS space (you do not need to verify that this is a geodesic), with the metric given by eq. (8.6) of the lectures, and set $\alpha = 1$. How does the path look in the coordinates $(\hat{t}, \hat{x})$ where
$$ds^2 = -d\hat{t}^2 + e^{2\hat{t}} d\hat{x}^2 \ ,$$
along the geodesic? (Give $\hat{t}(\tau), \hat{x}(\tau), \hat{x}(\hat{t})$ where τ the proper time of the particle.) What is the velocity of the particle at $\hat{t} = 0$ and at later times, as measured by an observer at rest in the $(\hat{t}, \hat{x})$ coordinates? Show that from $\hat{t} = -\infty$ to $\hat{t} = 0$ the particle has spent only a finite proper time in the patch $(\hat{t}, \hat{x})$. From where did it enter this patch?

2. **Geodesics in anti-de Sitter:** We introduced the AdS space as the 4-dimensional surface $-u^2 - v^2 + x^2 + y^2 + z^2 = -\alpha^2$ in a flat 5-d embedding space with the metric $ds^2 = -du^2 - dv^2 + dx^2 + dy^2 + dz^2$. In the coordinates (t', ρ, θ, ϕ) used in the lectures, the AdS metric becomes
$$ds^2 = \alpha^2 \left(-\cosh^2\!\rho \, dt'^2 + d\rho^2 + \sinh^2\!\rho \, (d\theta^2 + \sin^2\!\theta \, d\phi^2)\right) \ .$$
Use now another set of coordinates (t, χ, θ, ϕ) defined by
$$u = \alpha \sin(t/\alpha) \ , \quad v = \alpha \cos(t/\alpha) \cosh(\chi) \ , \quad x = \alpha \cos(t/\alpha) \sinh(\chi) \cos(\theta) \ ,$$
$$y = \alpha \cos(t/\alpha) \sinh(\chi) \sin(\theta) \cos(\phi) \ , \quad z = \alpha \cos(t/\alpha) \sinh(\chi) \sin(\theta) \sin(\phi) \ .$$
What is the metric in these new coordinates? What range of the entire manifold (t', ρ, θ, ϕ) is covered by the new coordinates? Use the fact that $(\chi, \theta, \phi) =$ const. is a geodesic to show that all timelike geodesics that beging from a given event P meet at another event Q.

3. **Milne univere:** Consider the RW metric
$$ds^2 = -dt^2 + a(t)^2 \left(\frac{dr^2}{1 - kr^2} + r^2 (d\theta^2 + \sin^2\!\theta \, d\phi^2) \right) \ ,$$
with $k = -1$ and $a = \pm t$ (this the solution of Friedmann equations for $\rho = p = 0$, i.e. empty space). Show that this represents a patch of the Minkowski space by finding the coordinate transfrom which converts the metric into
$$ds^2 = -dt'^2 + dr'^2 + r'^2 (d\theta^2 + \sin^2\!\theta \, d\phi^2) \ ,$$
(or vice verca). What part of the Minkowski space is covered? Draw a figure of the $t =$ const., $r =$ const. lines on the (t', r') plane.