General Relativity (FYSS7320), 2022 **Excercise 11.**

Return by 12.00, Wednesday 30.3.2022 (electronically to pyry.m.rahkila@jyu.fi).

1. **Gravitational waves from a binary star.** Consider a binary system where two neutron stars of mass $M = 2M_\odot$ orbit each other in a circular orbit such that their mutual distance is 10^6 km. Use Newtonian gravity to find the rotation period.

   a) Suppose the binary system is 1000 light year from us in the direction perpendicular to the orbit plane. What is the amplitude of the gravitational wave produced by the system as measured here? Express the result as the relative change in the distance between two freely falling test particles.

   b) As discussed in the lectures, the gravitational radiation reduces the energy of the system at the rate
   $$\frac{dE}{dt} = \frac{G}{5}\frac{d^3J_{ij}}{dt^3}\frac{d^3J^{ij}}{dt^3} \ .$$
   The velocities are small so that $I^{ij} = \int d^3x\, x^i x^j \rho$. What is the gravitational radiation power (in Watts) of the system? What is the power that goes through the Earth?

2. **Conformal diagram of the Minkowski space.** By a suitable coordinate transformation, an infinite spacetime can be mapped into a finite region. Consider as an example the Minkowski space for which the metric in spherical coordinates reads
   $$ds^2 = -dt^2 + dr^2 + r^2(d\theta^2 + \sin^2\theta\, d\phi^2) \ , \qquad -\infty < t < \infty \ , 0 \leqslant r < \infty \ .$$
   Perform the successive coordinate transformations
   $$\begin{aligned} u &= t - r \ , \qquad U = \arctan(u) \ , \qquad T = \frac{1}{2}(V + U) \ , \\ v &= t + r \ , \qquad V = \arctan(v) \ , \qquad R = \frac{1}{2}(V - U) \ . \end{aligned}$$

   a) What is the metric in coordinates T, R, θ, ϕ and what is the region of the R, T coordinates the Minkowski space corresponds to? Draw a diagram of it.

   b) Consider the world line of an object moving with a constant velocity $r = \beta t$ where $0 \leqslant t < \infty$. How does the world line appear in R, T coordinates for massive particles $0 < \beta < 1$ and light $\beta = 1$ (especially limits $t \to 0$ and $t \to \infty$)?

   c) Consider also the world line $R = \alpha T$ where $0 < \alpha < 1$. What is the limiting velocity dr/dt as $t \to \infty$?