Finite temperature field theory (FTFT) Excercise 4.

Return by 12.00, Monday 21.10.2021,
(electronically to olli.a.koskivaara@student.jyu.fi or in paper to a box outside Fys.1.)

1. Show explicitly the following identities

$$I_0(m) = \int \frac{\mathrm{d}^3\boldsymbol{p}}{(2\pi)^3}\frac{1}{2\omega} = \int \frac{\mathrm{d}^4 p}{(2\pi)^4}\frac{i}{k^2 - m^2}.$$

and

$$J_0(m) = -\int \frac{\mathrm{d}^3\boldsymbol{p}}{(2\pi)^3}\frac{\omega}{2} = i\int \frac{\mathrm{d}^4 p}{(2\pi)^4}\log(p^2 - m^2)$$

Consider these from the point of view of the contour integration and the dimensional regularization. Can you prove these also using the cut-off regularization?

4. Compute the full thermal integral

$$I(m) = \sum\!\!\!\!\!\!\!\int \frac{1}{p^2 + m_R^2} = \int \frac{\mathrm{d}^3\boldsymbol{p}}{(2\pi)^3}\frac{1}{m^2 + \boldsymbol{p}^2} + \sum\!\!\!\!\!\!\!\int{}' \frac{1}{p^2 + m^2}$$

where $\sum\!\!\!\!\!\!\int{}'$ refers to the sum without the zero-mode. Compute the zero-mode part and the high-frequency parts separately. Use both dimensional regularization and the cut-off (in 3d-space) regularization method. Show that in both cases performing you obtain the same divergent part as in excercise 1, even though the divergent parts are distributed differently between different frequency modes.

3. Consider a theory defined by the Lagrangian fucntion

$$\mathcal{L} = \sum_{i=1}^{2}\big[\frac{1}{2}(\partial_\mu\phi_i)^2 - \frac{m_i^2}{2}\phi_i^2 - \frac{\lambda}{4!}\phi_i^4\big] - g(\phi_1\phi_2^2 + \phi_2\phi_1^2).$$

Compute the thermal self-energy functions at one-loop level. Express the results explicitly in the high temperature limit $T \gg m_i$. Perform also the vacuum reonrmalization assuming that $\lambda_i, g > 0$, such that the theory has a symmetric vacuum state.