Finite temperature field theory (FTFT) Excercise 2.

Return by 12.00, Monday 27.9.2021,
(electronically to olli.a.koskivaara@student.jyu.fi or in paper to a box outside Fys.1.)

Excercise 1. Using $\hat{H} = \int \mathrm{d}^3\boldsymbol{x}(|\hat{\pi}|^2 + [\nabla\hat{\phi}|^2 + m^2|\hat{\phi}|^2)$ and $\hat{Q} = \int \mathrm{d}^3\boldsymbol{x}\ i(\hat{\phi}\pi - \hat{\phi}^*\hat{\pi}^*)$ for the Hamiltonian density and charge operators, show that $e^{-\beta\hat{H}}\hat{\phi}(\tau,\boldsymbol{x})e^{\beta\hat{H}} = \hat{\phi}(\tau-\beta,\boldsymbol{x})$ and $e^{\beta\mu\hat{Q}}\hat{\phi}e^{-\beta\mu\hat{Q}} = e^{-\beta\mu}\hat{\phi}$.

Excercise 2.how that the critical temperature for forming the condensate in the non-relativistic limit ($Q/V \ll m^3$, this is only an approximative condition) can be expressed as

$$T_c = \frac{2\pi}{m}\left(\frac{Q}{V}\right)^{2/3}\zeta(\tfrac{3}{2})^{-2/3}$$

and in the relativistic limit ($Q/V \gg m^3$) as

$$T_c = \Big(\frac{3}{m}\frac{Q}{V}\Big)^{1/2}.$$

Excercise 3. Show that the thermal part of the bosonic J_T^--function can be expanded as:

$$J_T^-(m,T) = -\frac{m^2T^2}{2\pi^2}\sum_{n=1}^{\infty}\frac{1}{n^2}K_2(\frac{nm}{T})$$

where $K_2(x)$ is the Bessel function of the second kind.

Excercise 4. Compute also the related integral:

$$I_T^-(m,T) \equiv \sum\!\!\!\!\!\!\int \Delta_0(\omega_n,\omega_{\boldsymbol{p}}) = \int\frac{\mathrm{d}^3\boldsymbol{p}}{(2\pi)^3}\frac{1}{2\omega}(1+2n_B(\omega_{\boldsymbol{p}})) = -\frac{mT}{2\pi^2}\sum_{n=1}^{\infty}\frac{1}{n}K_1(\frac{nm}{T}).$$

Show also that $mI_T^-(m,T) = \partial_m J_T^-(m,T)$.