JYVÄSKYLÄN YLIOPISTO
UNIVERSITY OF JYVÄSKYLÄ

Inverse problems with neutrinos

NCSU geometry and topology seminar

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August 31, 2022

Based on joint work with
Gunther Uhlmann

Topics

- The physics of neutrinos.
- Inverse problems with neutrinos.

Outline

Matter and force carriers

In the Standard Model of particle physics, there are two kinds of fundamental particles/fields:

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Note: Composite matter particles can have integer spin.

Matter in the Standard Model

There are two sectors of matter particles in SM:

Quarks

		electric charge	
		$q = -\frac{1}{3}e$	$q = +\frac{2}{3}e$
generation	1	d, down	u, up
	2	s, strange	c, charm
	3	b, bottom	t, top

Leptons

		electric charge	
		$q = -e$	$q = 0$
generation	1	e, electron	ν_1
	2	μ, muon	ν_2
	3	τ, tau	ν_3

Generations/flavors only differ by mass, and all 12 masses are different.
The charge jump is $1e$ in both sectors.

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The linear combination of the neutrinos ν_1, ν_2, ν_3 that couples to the electron is called the electron neutrino, $\nu_e = \sum_{i=1}^{3} U_{ei}\nu_i$.

Reminder

neutrino (neutral lepton)

$\neq$

neutron (udd)

Spacetime

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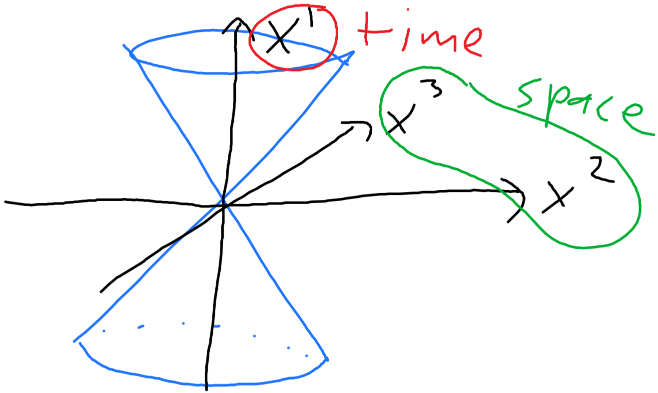
- General relativity lives in a Lorentzian manifold, and local GR is SR.

Spacetime



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Photons travel along light cones, massive particles inside the cones.

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This neutrino oscillation is a kinematic phenomenon.

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Kinematically we can thus treat neutrinos as perturbations of photons (which have $v = c$), so they are well modeled as *ultrarelativistic Jacobi fields along lightlike geodesics*.

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No other massive particle can travel across the universe.

Outline

1. Neutrino physics
2. Inverse problem 1: Medium effects on neutrino oscillations
   - Background: X-ray transforms
   - Neutrino oscillation
   - Measurement
   - The inverse problem
3. Inverse problem 2: Breaking conformal gauge

Background: X-ray transforms

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The solution operator $C_f([0, L])$ taking an initial $I(0) \in \mathbb{R}^N$ to the final $I(L) \in \mathbb{R}^N$ depending on the function $f : \Omega \to \mathbb{R}^{N \times N}$ is called the non-Abelian X-ray transform of f.

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This transform is injective: $C_f(\gamma)$ for all lines γ determines the function f.

If everything commutes, then

$$C_f(\gamma) = \exp\left(-\int f(\gamma(t))\mathrm{d}t\right)$$

as a matrix.

Neutrino oscillation

The state space of a neutrino is 3-dimensional, and the state (in the flavor basis) is

$$\psi(t) = \begin{pmatrix} \psi_e(t) \\ \psi_\mu(t) \\ \psi_\tau(t) \end{pmatrix} \in \mathbb{C}^3.$$

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Semiclassical description: A classical point particle that carries a quantum state.

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In vacuum we have just the free Hamiltonian

$$H_0 = \frac{1}{2E} U_{\text{PMNS}} \begin{pmatrix} m_1^2 & 0 & 0 \\ 0 & m_2^2 & 0 \\ 0 & 0 & m_3^2 \end{pmatrix} U_{\text{PMNS}}^*.$$

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In a medium we have $H = H_0 + N_e A$, where

$$A = 2\sqrt{2} E G_F \begin{pmatrix} 1 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix},$$

where N_e is the electron number density.

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Phase information is lost, so multiples of the identity matrix are invisible.

The inverse problem

Question

If we can prepare and measure the initial and final states of neutrinos passing through an object, can we find the electron number density $N_e(x)$?

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Theorem [I., 2016]

Suppose we do the measurements above for all straight lines through a nice domain $\Omega \subset \mathbb{R}^n$ with $n \geq 2$ with a hermitean Hamiltonian field $H: \Omega \to \mathbb{C}^{n \times n}$.

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If the measurement library is big enough, then the measurements determine the trace-free part of $H(x)$ uniquely for all $x \in \Omega$.

The conclusion is more than strong enough for the physical problem, but we also assume too big a library.

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(Only one type of neutrino in this problem.)

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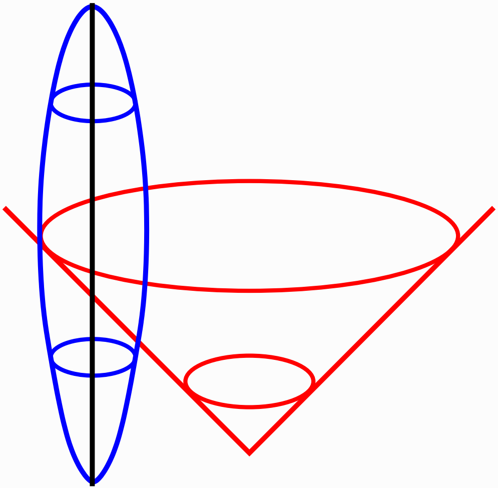
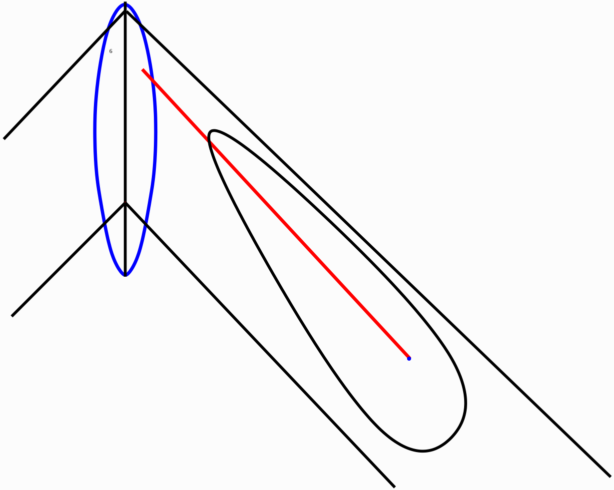


Light cone from a source measured in an observation set.

The goal



We can look back along lightlike geodesics.
The union of these geodesics makes up our visible past.

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Neutrino worldlines are photon wordlines plus an infinitesimal variation.
They break the conformal symmetry infinitesimally and are sensitive to the conformal factor.

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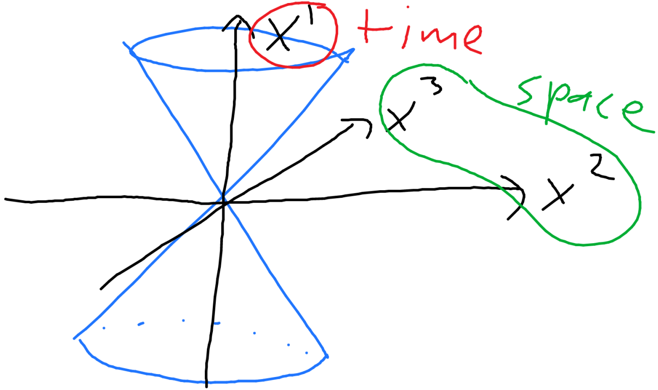

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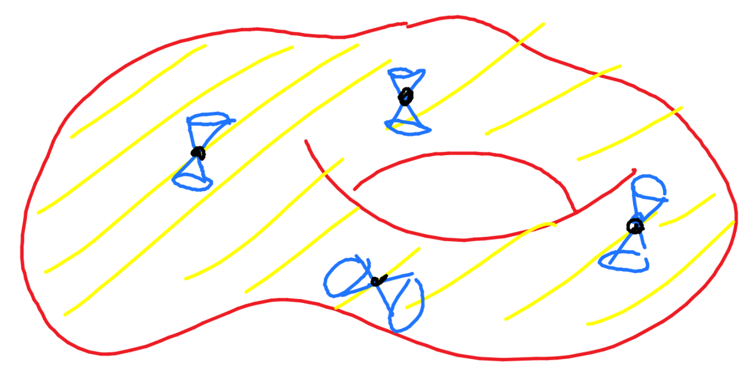
- Particles travelling at the speed of light only care about the conformal class. They have no sense of scale, local or global!
- Mass is the ability to sense scale.

Conformal gauge

light cone bundle $= \{v \in TM; v \cdot v = 0\}$

$\leftrightarrow$ possible photon directions

$\leftrightarrow$ conformal class



The result

Theorem (Kurylev–Lassas–Uhlmann, 2018)

Measurements of light cones in an open subset of the spacetime determine the geometry and conformal class of the spacetime in the lightlike past of the measurement set.

Measurements of photons determine everything in the visible part of the spacetime except except the conformal factor: We cannot tell (M, g) and (M, cg) apart.

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Measurements of light cones in an open subset of the spacetime determine the geometry and conformal class of the spacetime in the lightlike past of the measurement set.

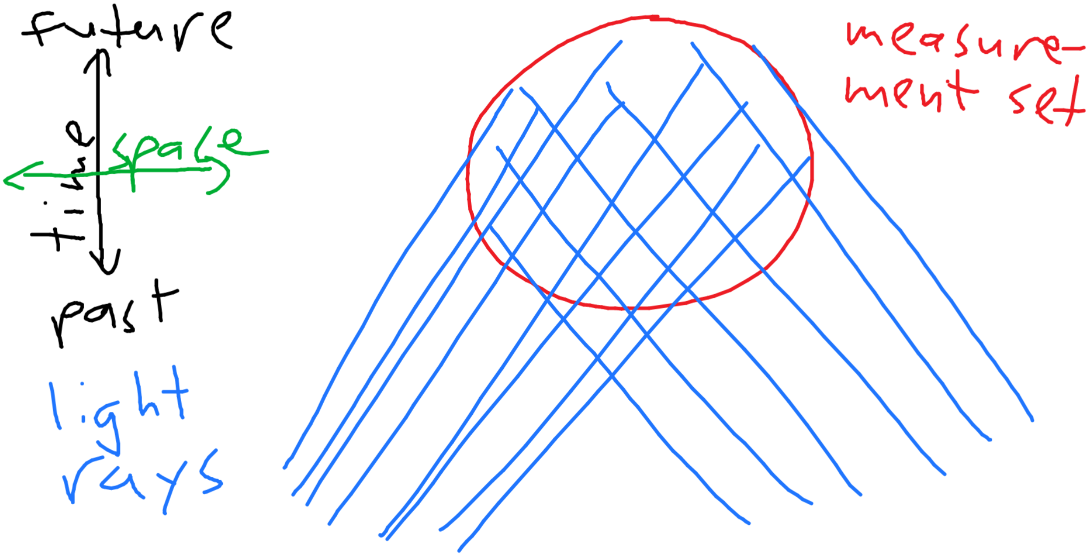
Measurements of photons determine everything in the visible part of the spacetime except except the conformal factor: We cannot tell (M, g) and (M, cg) apart.

Theorem (I.–Uhlmann, 2021)

Suppose the conformal class is known. Measurements of (perturbative) neutrino cones in an open subset of the spacetime determine the conformal factor in the lightlike past of the measurement set.

Photons and neutrinos together determine the full geometry of the visible part of the spacetime!

Visible past



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- Neutrinos are a tiny bit slower than photons ($v \approx (1 - 10^{-20})c$) but are released a little earlier.
- The neutrino cone depends on the motion of the dying star, the light cone does not.
- Supernovae are fairly dense in the spacetime on cosmological scales: If we measure for a year and see a distance of 10^{10} lightyears, then the visible part of the spacetime has 10^{10} supernovae.

The two cones

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- What we measure is not deviations of light rays, but deviations of light cones.
  $\implies$ We need to take the component of the ultrarelativistic Jacobi field normal to the light cone.
- If $\gamma(t)$ is the light ray, the direction normal to the light cone is $\dot{\gamma}(t)$.

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  $\implies$ The inverse problem becomes simple!

Lessons

Neutrinos. . .

- . . . interact very weakly in all respects.
- . . . oscillate between the flavors ν_e, ν_μ, ν_τ.
- . . . have a tiny mass, but that is enough to sense scale and fix a conformal factor.
- . . . are well modeled by ultrarelativistic Jacobi fields along light rays.
- . . . can see what other particles cannot.
- . . . are fun.

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