JYVÄSKYLÄN YLIOPISTO
UNIVERSITY OF JYVÄSKYLÄ

Breaking cosmological conformal gauge with neutrinos

Inverse Days

Joonas Ilmavirta

December 15, 2021

Based on joint work with
Gunther Uhlmann

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How can we break this conformal symmetry?

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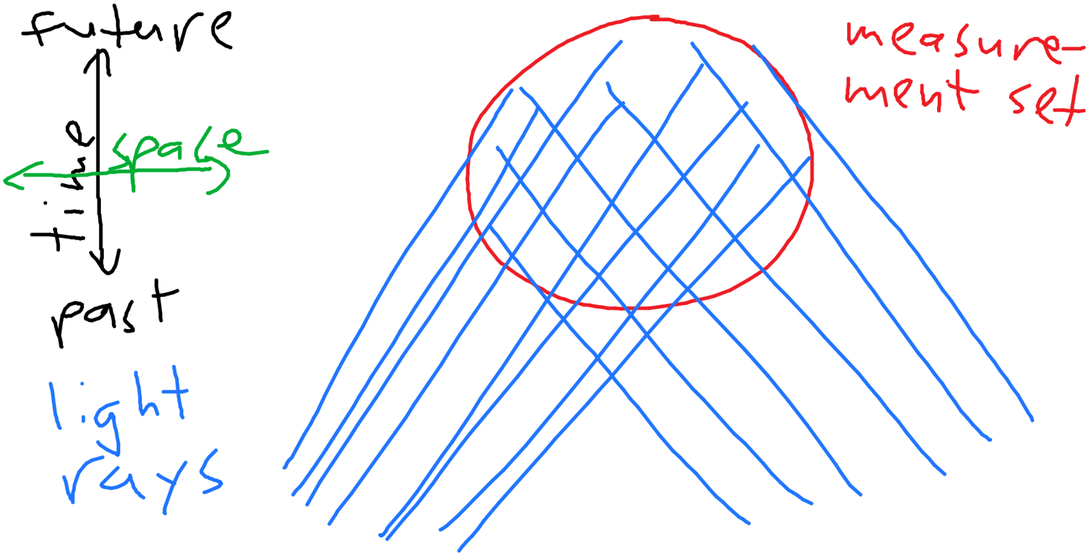
Photons and neutrinos together determine the full geometry of the visible part of the spacetime!

Visible past



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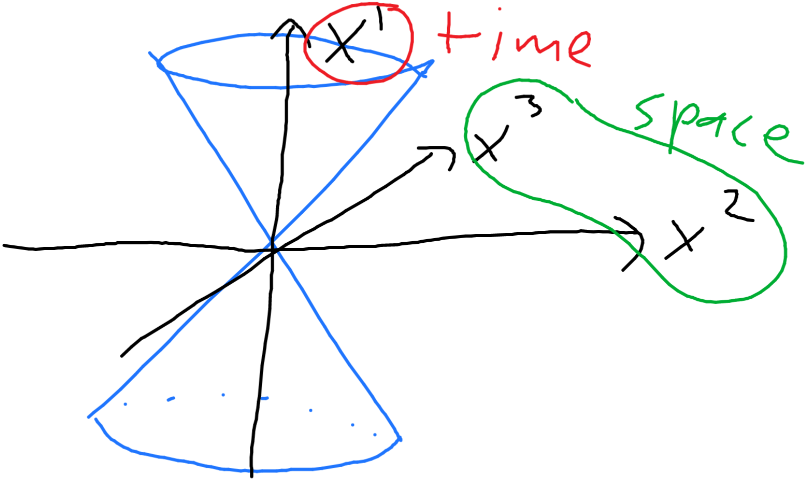
- General relativity lives in a Lorentzian manifold, and local GR is SR.

Light cone



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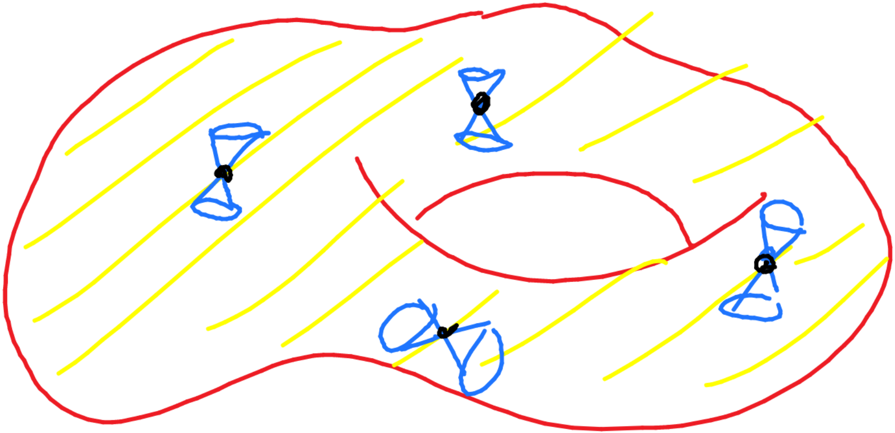

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Neutrinos have tiny but non-zero mass and are typically ultrarelativistic. They are electrically neutral and interact very weakly but can be observed with specialized detectors. They are produced in great numbers in supernova explosions; about 99 % of supernova energy is carried by neutrinos.

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- The neutrino cone depends on the motion of the dying star, the light cone does not.

The two cones

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  $\implies$ The inverse problem becomes simple!

A couple of lessons

- Supernova photons determine everything but a conformal factor.
- Mass is the ability to sense scale, so a massive particle is needed to fix a conformal factor.
- Neutrinos have a tiny mass, but it is enough to break the conformal symmetry.
- Ultrarelativistic particles can be modeled by Jacobi fields along light rays.

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Ask for details:

joonas.ilmavirta@jyu.fi