JYVÄSKYLÄN YLIOPISTO
UNIVERSITY OF JYVÄSKYLÄ

Spectral and travel time tomography on Mars with InSight

Applied Inverse Problems minisymposium
How to see inside the Earth? Theory and applications of seismic inverse problems

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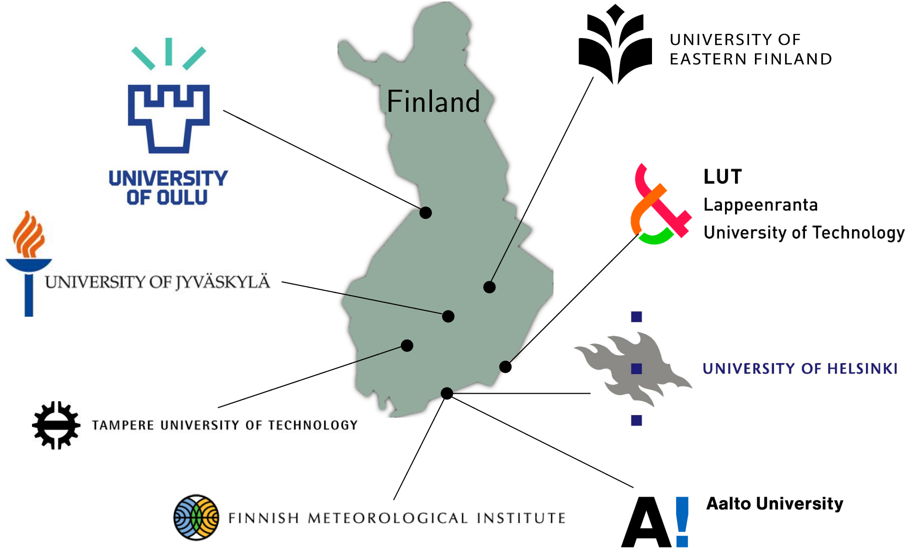
July 12, 2019

Based on joint work with
Maarten de Hoop and Vitaly Katsnelson

Finnish Centre of Excellence in Inverse Modelling and Imaging

2018-2025

CENTRES OF EXCELLENCE IN RESEARCH



Conference announcement

The annual Finnish inverse problems conference “Inverse Days” will be organized in Jyväskylä 16–18 December, 2019.

`http://r.jyu.fi/yVK`

(`https://www.jyu.fi/science/en/maths/research/inverse-problems/id2019/`)

Registration opens this week!

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- Assume perfect measurements from a single ideal seismometer. What can you say for sure and is there an inversion algorithm?
- Identifying useful data sets can help future mission planning.
- Grand goal: A mathematical theory of seismic planetary exploration.

Outline

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- Mars is roughly spherically symmetric. There are reliable ways to reconstruct a radial model of the (upper) mantle from a single station. (The mantle determines the CMB.)
- I will ignore noise, model errors, finiteness, stability, and many other practical things.

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- Linearized data: Pairs of periodic broken rays and integrals over them. Uknown: Variations of wave speed (a function).

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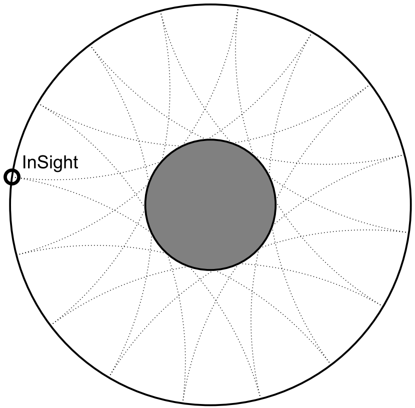

Periodic seismic ray reflecting on the surface and CMB.

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(Uniqueness should be provable for the non-linear one, too.)

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- Mathematically, the spectrum of free oscillations corresponds to the Neumann spectrum of the Laplace–Beltrami operator on a manifold.

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If a family of wave speeds $c_s(r)$ have the same spectrum, are the equal? Is the (Martian) mantle spectrally rigid?

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This is based on linearization. With a trace formula one ends up showing that the Martian mantle is length spectrally rigid.

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- Surface waves will come from the event to InSight two ways along the great circle containing the impact site and InSight.
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- Multiple arrivals or a priori information tells the time T around the great circle.

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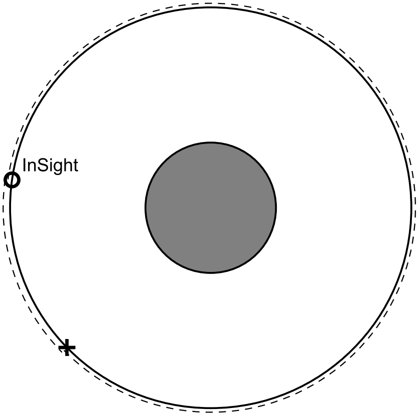

Two surface wave arrivals from the same event.

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- To get here, we needed to assume spherical symmetry only on the surface, but the arising problem is easiest to solve if the symmetry extends inside.

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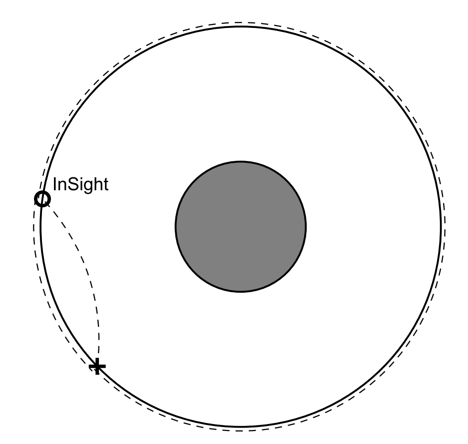

The body wave whose initial point and time were located with surface waves.

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- The linearized problem is X-ray tomography (or an Abel transform), and can also be solved explicitly. (e.g. de Hoop–I., 2017)

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- This gives us an isotropic radially symmetric reference model of the mantle, which is a stepping stone towards deeper and finer structure.

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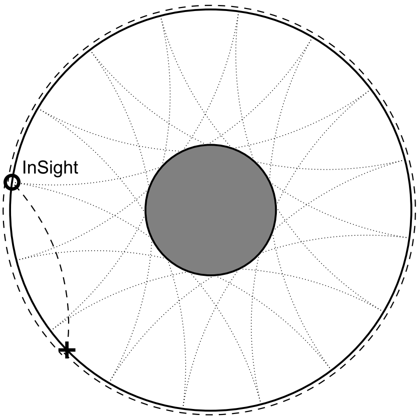

Three ways to see the mantle from InSight.

Summary

- A: From noise correlations to (linearized) travel times.
- B: From spectrum to length spectrum.
- C: Meteorites; body wave data calibrated by surface waves.

Outline

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- It remains to be proven that this is indeed the only obstruction to uniqueness.

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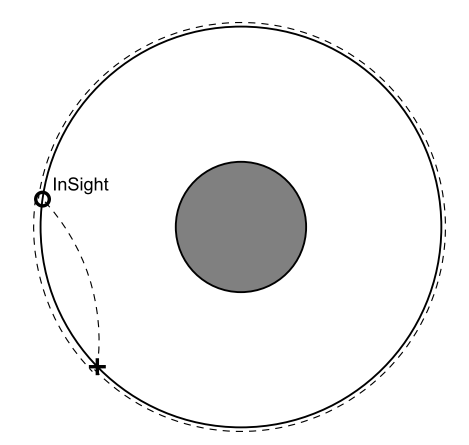
- This leads to travel time data: The travel times (geometrically: distances) are known from all points on the surface to a single fixed point.

Half-local X-ray tomography



The body wave whose initial point and time were located with surface waves.

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What if we linearize the problem? (X-ray tomography.)

This is possible at least in Euclidean geometry or with real analytic perturbations but always unstable.

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- What is the geometry of periodic broken rays?

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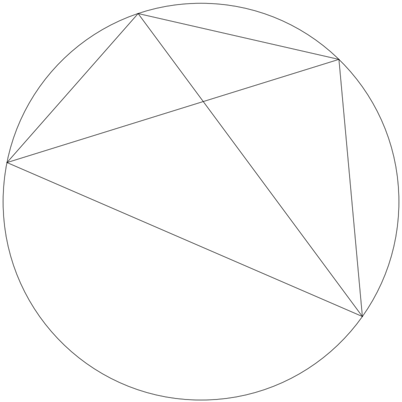
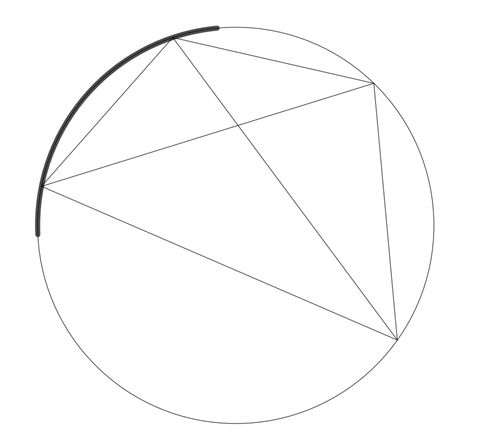


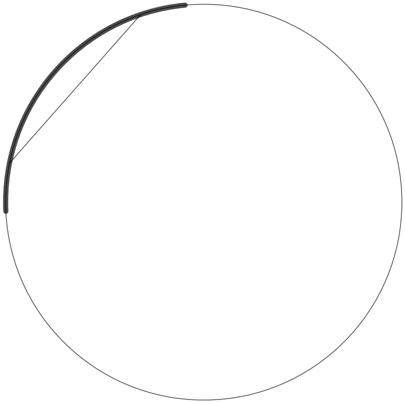
Boundary distance rigidity: Do the distances between all boundary points determine the meometry?

A theory?



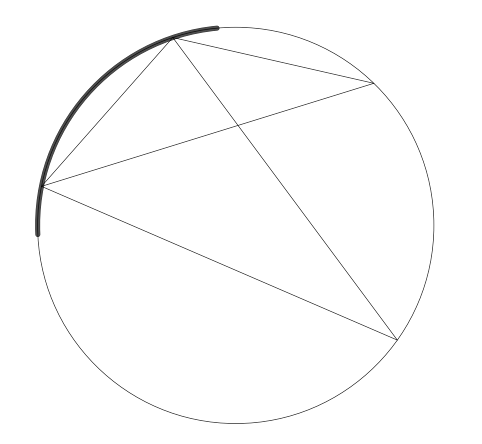
We have an accessible region — a measurement array.
The size is exaggerated.

A theory?



In the local boundary distance problem one knows the distances between the points in the small set and wants to find the geometry near that set.

A theory?



The “half-local” boundary distance data has more information and one wants to reconstruct the whole geometry.

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