JYVÄSKYLÄN YLIOPISTO
UNIVERSITY OF JYVÄSKYLÄ

Geometrization, inverse problems, and physics

Jyväskylä physics colloquium

Joonas Ilmavirta

September 23, 2022

Outline

Inverse problems

Many things have to be measured indirectly:

- What is inside the Earth?
- Is the concrete beam reliable?
- Has my bone fractured?
- And many more...

Inverse problems

Many things have to be measured indirectly:

- What is inside the Earth?
- Is the concrete beam reliable?
- Has my bone fractured?
- And many more…

Direct problem: Given the cause, find the effect.

Inverse problems

Many things have to be measured indirectly:

- What is inside the Earth?
- Is the concrete beam reliable?
- Has my bone fractured?
- And many more. . .

Direct problem: Given the cause, find the effect.

Inverse problem: Given the effect, find the cause.

Inverse problems

Many things have to be measured indirectly:

- What is inside the Earth?
- Is the concrete beam reliable?
- Has my bone fractured?
- And many more. . .

Direct problem: Given the cause, find the effect.

Inverse problem: Given the effect, find the cause.

There are many interpretations of the word “find”.

X-ray tomography

Consider a beam of light travelling right on the real axis. The Beer–Lambert law says that the intensity satisfies

$$I'(x) = -\mu(x)I(x),$$

where $\mu(x)$ is the attenuation coefficient.

X-ray tomography

Consider a beam of light travelling right on the real axis. The Beer–Lambert law says that the intensity satisfies

$$I'(x) = -\mu(x)I(x),$$

where $\mu(x)$ is the attenuation coefficient.

If the ray starts at $x = 0$ and ends at $x = L$, we have

$$I(L) = I(0)\exp\left(-\int_0^L \mu(x)\mathrm{d}x\right).$$

X-ray tomography

Consider a beam of light travelling right on the real axis. The Beer–Lambert law says that the intensity satisfies

$$I'(x) = -\mu(x)I(x),$$

where $\mu(x)$ is the attenuation coefficient.

If the ray starts at $x = 0$ and ends at $x = L$, we have

$$I(L) = I(0)\exp\left(-\int_0^L \mu(x)\mathrm{d}x\right).$$

X-ray images in all directions $\iff$ all line integrals of μ.

X-ray tomography

Direct problem: Given a function $\mu(x)$ (positive function in $\mathbb{R}^2$ or $\mathbb{R}^3$ with compact support), compute all its line integrals.

X-ray tomography

Direct problem: Given a function $\mu(x)$ (positive function in $\mathbb{R}^2$ or $\mathbb{R}^3$ with compact support), compute all its line integrals.

Inverse problem: Given all the line integrals of a function (the X-ray transform of the function), find the function.

X-ray tomography

Direct problem: Given a function $\mu(x)$ (positive function in $\mathbb{R}^2$ or $\mathbb{R}^3$ with compact support), compute all its line integrals.

Inverse problem: Given all the line integrals of a function (the X-ray transform of the function), find the function.

Many questions arise:

- Uniqueness: Is the unknown uniquely determined by the data?

X-ray tomography

Direct problem: Given a function $\mu(x)$ (positive function in $\mathbb{R}^2$ or $\mathbb{R}^3$ with compact support), compute all its line integrals.

Inverse problem: Given all the line integrals of a function (the X-ray transform of the function), find the function.

Many questions arise:

- Uniqueness: Is the unknown uniquely determined by the data?
- Algorithm: If yes, is there a way to compute the function from the data?

X-ray tomography

Direct problem: Given a function $\mu(x)$ (positive function in $\mathbb{R}^2$ or $\mathbb{R}^3$ with compact support), compute all its line integrals.

Inverse problem: Given all the line integrals of a function (the X-ray transform of the function), find the function.

Many questions arise:

- Uniqueness: Is the unknown uniquely determined by the data?
- Algorithm: If yes, is there a way to compute the function from the data?
- Stability: How sensitive is the reconstruction to measurement errors?

X-ray tomography

Direct problem: Given a function $\mu(x)$ (positive function in $\mathbb{R}^2$ or $\mathbb{R}^3$ with compact support), compute all its line integrals.

Inverse problem: Given all the line integrals of a function (the X-ray transform of the function), find the function.

Many questions arise:

- Uniqueness: Is the unknown uniquely determined by the data?
- Algorithm: If yes, is there a way to compute the function from the data?
- Stability: How sensitive is the reconstruction to measurement errors?
- Range: What can valid data look like?

X-ray tomography

Johann Radon proved uniqueness and gave an inversion formula in 1917. He saw it as a purely mathematical problem.

X-ray tomography

Johann Radon proved uniqueness and gave an inversion formula in 1917. He saw it as a purely mathematical problem.

Allan Cormack was thinking of the X-ray tomography application and resolved the problem in a different way in 1963. He did not know of Radon's work.

X-ray tomography

Johann Radon proved uniqueness and gave an inversion formula in 1917. He saw it as a purely mathematical problem.

Allan Cormack was thinking of the X-ray tomography application and resolved the problem in a different way in 1963. He did not know of Radon's work.

Godfrey Hounsfield built an X-ray scanner in 1971.

X-ray tomography

Johann Radon proved uniqueness and gave an inversion formula in 1917. He saw it as a purely mathematical problem.

Allan Cormack was thinking of the X-ray tomography application and resolved the problem in a different way in 1963. He did not know of Radon's work.

Godfrey Hounsfield built an X-ray scanner in 1971.

Cormack and Hounsfield were awarded the Nobel Prize in Physiology or Medicine in 1979.

X-ray tomography

Johann Radon proved uniqueness and gave an inversion formula in 1917. He saw it as a purely mathematical problem.

Allan Cormack was thinking of the X-ray tomography application and resolved the problem in a different way in 1963. He did not know of Radon's work.

Godfrey Hounsfield built an X-ray scanner in 1971.

Cormack and Hounsfield were awarded the Nobel Prize in Physiology or Medicine in 1979.

Lesson: We would have gotten this lifesaving technology faster if physicists and mathematicians had been aware of each other's problems and results.

A spectrum of inverse problems

There are many aspects of inverse problems:

- Mathematical modelling: Ask the right questions.
- Pure mathematics: Is the indirect measurement possible with ideal data even in principle?
- Applied mathematics: How does the practical goal and our a priori knowledge change how the problem should be solved?
- Numerical analysis: How to efficiently and accurately compute the unknown from the data?
- Measurements: What and how to measure usefully and with minimal harm?

A spectrum of inverse problems

There are many aspects of inverse problems:

- Mathematical modelling: Ask the right questions.
- Pure mathematics: Is the indirect measurement possible with ideal data even in principle?
- Applied mathematics: How does the practical goal and our a priori knowledge change how the problem should be solved?
- Numerical analysis: How to efficiently and accurately compute the unknown from the data?
- Measurements: What and how to measure usefully and with minimal harm?

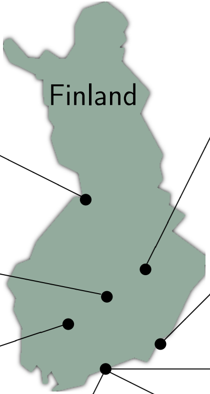
These are all represented in the Centre of Excellence of Inverse Modelling and Imaging.

The inverse problems group at Jyväskylä does mostly pure mathematics and a bit of modelling.

Finnish Centre of Excellence in Inverse Modelling and Imaging

2018-2025

CENTRES OF EXCELLENCE IN RESEARCH



UNIVERSITY OF OULU

UNIVERSITY OF JYVÄSKYLÄ

Tampere University

FINNISH METEOROLOGICAL INSTITUTE

UNIVERSITY OF EASTERN FINLAND

LUT
Lappeenranta
University of Technology

UNIVERSITY OF HELSINKI

Aalto University

Want to hear more?

Nationally:
The annual Finnish inverse problems conference Inverse Days is at Tahko December 12–16, 2022. The registration is open now.

Locally:
Informal meetings at Jyväskylä?

Outline

1. Inverse problems I: Introduction
2. Geometrization I: Wave–particle duality
   - Hyperbolic PDEs
   - Smoothness of solutions
   - Singularities
   - Propagation of singularities
   - Microlocal analysis
3. Geometrization II: Everything goes straight
4. Inverse problems II: Geomathematics
5. Summary

Hyperbolic PDEs

A typical classification of PDEs (partial differential equations):

- Elliptic equations like the Laplace equation $\nabla^2 u = 0$.
- Parabolic equations like the diffusion equation $\partial_t u = \kappa \nabla^2 u$.
- Hyperbolic equations like the wave equation $\partial_t^2 u - c^2 \nabla^2 u = 0$.
- Other equations like the Schrödinger equation $i\hbar \partial_t u = -\frac{\hbar^2}{2m} \nabla^2 u$.

Hyperbolic PDEs

A typical classification of PDEs (partial differential equations):

- Elliptic equations like the Laplace equation $\nabla^2 u = 0$.
- Parabolic equations like the diffusion equation $\partial_t u = \kappa \nabla^2 u$.
- Hyperbolic equations like the wave equation $\partial_t^2 u - c^2 \nabla^2 u = 0$.
- Other equations like the Schrödinger equation $i\hbar \partial_t u = -\frac{\hbar^2}{2m} \nabla^2 u$.

Solutions to hyperbolic equations can be called waves — the elliptic and parabolic ones do not have a wave-like behaviour in a technical sense.

Smoothness of solutions

Smooth = infinitely differentiable, C^∞.

Smoothness of solutions

Smooth $=$ infinitely differentiable, C^∞.

If $u(x)$ solves an elliptic equation in some open set of the space, then $u(x)$ is smooth in that set.

Smoothness of solutions

Smooth $=$ infinitely differentiable, C^∞.

If $u(x)$ solves an elliptic equation in some open set of the space, then $u(x)$ is smooth in that set.

If $u(t, x)$ solves a parabolic equation in some open set of the spacetime, then $u(t, x)$ is smooth in that set.

Smoothness of solutions

Smooth $=$ infinitely differentiable, C^∞.

If $u(x)$ solves an elliptic equation in some open set of the space, then $u(x)$ is smooth in that set.

If $u(t,x)$ solves a parabolic equation in some open set of the spacetime, then $u(t,x)$ is smooth in that set.

A solution to a hyperbolic equation does not have to be smooth.
This turns out to be useful!

Singularities

A solution to a hyperbolic equation can be a non-smooth distribution.

Singularities

A solution to a hyperbolic equation can be a non-smooth distribution.

There are different ways to define singularities of a function $u(t, x)$:

- A point where some derivative of u blows up.

Singularities

A solution to a hyperbolic equation can be a non-smooth distribution.

There are different ways to define singularities of a function $u(t,x)$:

- A point where some derivative of u blows up.
- The wave-front set WFu is the set of points (x,v) so that u has a singularity at x in the direction of v.
  This is defined via cut-offs and Fourier analysis.

Singularities

A solution to a hyperbolic equation can be a non-smooth distribution.

There are different ways to define singularities of a function $u(t, x)$:

- A point where some derivative of u blows up.
- The wave-front set WFu is the set of points (x, v) so that u has a singularity at x in the direction of v.
  This is defined via cut-offs and Fourier analysis.

Idea: Ignore the smooth part of your data and only look at the singularities.

Singularities

A solution to a hyperbolic equation can be a non-smooth distribution.

There are different ways to define singularities of a function $u(t,x)$:

- A point where some derivative of u blows up.
- The wave-front set WFu is the set of points (x,v) so that u has a singularity at x in the direction of v.
  This is defined via cut-offs and Fourier analysis.

Idea: Ignore the smooth part of your data and only look at the singularities.
You lose some information, but what remains is more tractable.

Propagation of singularities

Microlocal analysis provides an explicit description of how these singularities move:
The propagation of singularities is described by a Hamiltonian flow whose Hamiltonian can be computed from the PDE.

Propagation of singularities

Microlocal analysis provides an explicit description of how these singularities move: The propagation of singularities is described by a Hamiltonian flow whose Hamiltonian can be computed from the PDE.

This is a mathematician's wave–particle duality: The treatment goes from waves (solutions of a hyperbolic equation) to point particles (singularities).

Propagation of singularities

Microlocal analysis provides an explicit description of how these singularities move:
The propagation of singularities is described by a Hamiltonian flow whose Hamiltonian can be computed from the PDE.

This is a mathematician's wave–particle duality: The treatment goes from waves (solutions of a hyperbolic equation) to point particles (singularities).

Benefits:

- Focus first on leading order or high-frequency behaviour.
- The singularities behave better than the smooth part of the solution.
  There is an explicit and nice geometric description of their motion.

Microlocal analysis

An assortment of ideas from microlocal analysis:

Microlocal analysis

An assortment of ideas from microlocal analysis:

- A partial differential operator only sees the local values of a function.
  A pseudodifferential operator can be non-local but it treats singularities locally.

Microlocal analysis

An assortment of ideas from microlocal analysis:

- A partial differential operator only sees the local values of a function.
  A pseudodifferential operator can be non-local but it treats singularities locally.
- Fourier integral operators move singularities in controlled ways.

Microlocal analysis

An assortment of ideas from microlocal analysis:

- A partial differential operator only sees the local values of a function.
  A pseudodifferential operator can be non-local but it treats singularities locally.
- Fourier integral operators move singularities in controlled ways.
- The principal symbol captures the leading order behaviour of a PDE and determines the propagation of singularities.
  The symbol of the energy operator $V(x) - \frac{\hbar^2}{2m}\nabla^2$ is the total energy $V(x) + \frac{p^2}{2m}$.

Microlocal analysis

An assortment of ideas from microlocal analysis:

- A partial differential operator only sees the local values of a function.
  A pseudodifferential operator can be non-local but it treats singularities locally.
- Fourier integral operators move singularities in controlled ways.
- The principal symbol captures the leading order behaviour of a PDE and determines the propagation of singularities.
  The symbol of the energy operator $V(x) - \frac{\hbar^2}{2m}\nabla^2$ is the total energy $V(x) + \frac{p^2}{2m}$.
- Going from a symbol to an operator is called quantization.
  One quantization of the quantity $\omega^2 - c^2 |k|^2$ (frequency ω and wave vector k) is the wave operator $\partial_t^2 - c^2\nabla^2$.

Microlocal analysis

An assortment of ideas from microlocal analysis:

- A partial differential operator only sees the local values of a function.
  A pseudodifferential operator can be non-local but it treats singularities locally.
- Fourier integral operators move singularities in controlled ways.
- The principal symbol captures the leading order behaviour of a PDE and determines the propagation of singularities.
  The symbol of the energy operator $V(x) - \frac{\hbar^2}{2m}\nabla^2$ is the total energy $V(x) + \frac{p^2}{2m}$.
- Going from a symbol to an operator is called quantization.
  One quantization of the quantity $\omega^2 - c^2 |k|^2$ (frequency ω and wave vector k) is the wave operator $\partial_t^2 - c^2\nabla^2$.
- Roughly: Quantization and taking symbols moves between a classical and a quantum description.

Outline

Gravitation

In Newton's gravitation the Sun exerts a force that causes the Earth to travel along a curved path.
(Gravity is a force.)

Gravitation

In Newton's gravitation the Sun exerts a force that causes the Earth to travel along a curved path.
(Gravity is a force.)

In Einstein's gravitation the Sun causes the geometry of the spacetime to curve, and the Earth travels straight in this curved geometry.
(Gravity is an interaction between geometry and matter.)

Gravitation

In Newton's gravitation the Sun exerts a force that causes the Earth to travel along a curved path.
(Gravity is a force.)

In Einstein's gravitation the Sun causes the geometry of the spacetime to curve, and the Earth travels straight in this curved geometry.
(Gravity is an interaction between geometry and matter.)

The same idea can be adopted to many situations:
Declare a new geometry so that the relevant curves are all straight (= geodesics).
This might not bring anything new phenomenologically, but it changes the point of view and gives access to new tools.

Fermat's principle

The most fundamental concept of geometry is distance. All other things (area, angle, curvature, length, straightness, . . .) can be derived from it.

Fermat's principle

The most fundamental concept of geometry is distance. All other things (area, angle, curvature, length, straightness, . . .) can be derived from it.

There are two natural distances between any two points x and y:

- The spatial distance $d_S(x, y)$ measured in meters.
- The temporal distance $d_T(x, y)$ measured in seconds.
  (This is the time it takes for waves to travel from x to y.)

Fermat's principle

The most fundamental concept of geometry is distance. All other things (area, angle, curvature, length, straightness, ...) can be derived from it.

There are two natural distances between any two points x and y:

- The spatial distance $d_S(x, y)$ measured in meters.
- The temporal distance $d_T(x, y)$ measured in seconds. (This is the time it takes for waves to travel from x to y.)

Fermat's principle: The wave goes straight (along a geodesic) in the temporal geometry.

Fermat's principle

The most fundamental concept of geometry is distance. All other things (area, angle, curvature, length, straightness, ...) can be derived from it.

There are two natural distances between any two points x and y:

- The spatial distance $d_S(x, y)$ measured in meters.
- The temporal distance $d_T(x, y)$ measured in seconds.
  (This is the time it takes for waves to travel from x to y.)

Fermat's principle: The wave goes straight (along a geodesic) in the temporal geometry.

Geodesics minimize short distances but not necessarily long ones.
Therefore Fermat's principle is not about minimizing time but about going along a geodesic in the temporal geometry.

Optics

What happens in gravitation happens in all kinds of optics:

- Traditional view: Variations of sound speed cause the waves to curve.
- Geometric view: Variations of sound speed define a geometry where all waves go straight.

Optics

What happens in gravitation happens in all kinds of optics:

- Traditional view: Variations of sound speed cause the waves to curve.
- Geometric view: Variations of sound speed define a geometry where all waves go straight.

If speed of sound is $c(x)$, then the temporal geometry is Riemannian with the metric tensor $c^{-2}\mathrm{d}x^2$.

Optics

What happens in gravitation happens in all kinds of optics:

- Traditional view: Variations of sound speed cause the waves to curve.
- Geometric view: Variations of sound speed define a geometry where all waves go straight.

If speed of sound is $c(x)$, then the temporal geometry is Riemannian with the metric tensor $c^{-2}\mathrm{d}x^2$.

This changes the inverse problem:

- Traditional view: Given the data, find the variable sound speed $c(x)$.
- Geometric view: Given the data, find the (Riemannian or other) geometry.

Different kinds of geometries

Different settings lead to different kinds of geometries:

- In a Riemannian geometry an inner product is defined at every point.

Different kinds of geometries

Different settings lead to different kinds of geometries:

- In a Riemannian geometry an inner product is defined at every point.
- In Finsler geometry a norm is defined at every point.

Different kinds of geometries

Different settings lead to different kinds of geometries:

- In a Riemannian geometry an inner product is defined at every point.
- In Finsler geometry a norm is defined at every point.
- Sometimes we do not even get a norm, but only a Hamiltonian flow.

Different kinds of geometries

Different settings lead to different kinds of geometries:

- In a Riemannian geometry an inner product is defined at every point.
- In Finsler geometry a norm is defined at every point.
- Sometimes we do not even get a norm, but only a Hamiltonian flow.
- In general relativity there is something resembling an inner product at every event.

Different kinds of geometries

Different settings lead to different kinds of geometries:

- In a Riemannian geometry an inner product is defined at every point.
- In Finsler geometry a norm is defined at every point.
- Sometimes we do not even get a norm, but only a Hamiltonian flow.
- In general relativity there is something resembling an inner product at every event.

If there are different polarizations, they can have different geometries, each with their own speeds and straight paths.

Different kinds of geometries

Different settings lead to different kinds of geometries:

- In a Riemannian geometry an inner product is defined at every point.
- In Finsler geometry a norm is defined at every point.
- Sometimes we do not even get a norm, but only a Hamiltonian flow.
- In general relativity there is something resembling an inner product at every event.

If there are different polarizations, they can have different geometries, each with their own speeds and straight paths.

$\rightsquigarrow$ Multiple geometries!

Two steps to a geometric theory

Step 1: Replace waves with point particles.

Step 2: Encode the dynamics or material parameters into a such a geometry that the point particles go straight.

Outline

The model

Parameters:

- Stiffness tensor $c_{ijkl}(x)$, the “spring constant” of the medium.
- Density $\rho(x)$.

The model

Parameters:

- Stiffness tensor $c_{ijkl}(x)$, the “spring constant” of the medium.
- Density $\rho(x)$.

The field: Displacement $u(t,x) \in \mathbb{R}^3$ from equilibrium.

The model

Parameters:

- Stiffness tensor $c_{ijkl}(x)$, the "spring constant" of the medium.
- Density $\rho(x)$.

The field: Displacement $u(t,x) \in \mathbb{R}^3$ from equilibrium.

The elastic wave equation:

$$\rho(x)\partial_t^2 u_i(t,x) - \sum_{j,k,l} \partial_j [c_{ijkl}(x)\partial_k u_l(t,x)] = 0.$$

The model

Parameters:

- Stiffness tensor $c_{ijkl}(x)$, the “spring constant” of the medium.
- Density $\rho(x)$.

The field: Displacement $u(t,x) \in \mathbb{R}^3$ from equilibrium.

The elastic wave equation:

$$\rho(x)\partial_t^2 u_i(t,x) - \sum_{j,k,l} \partial_j[c_{ijkl}(x)\partial_k u_l(t,x)] = 0.$$

The stiffness tensor can be anisotropic (21 components).

The model

Parameters:

- Stiffness tensor $c_{ijkl}(x)$, the “spring constant” of the medium.
- Density $\rho(x)$.

The field: Displacement $u(t,x) \in \mathbb{R}^3$ from equilibrium.

The elastic wave equation:

$$\rho(x)\partial_t^2 u_i(t,x) - \sum_{j,k,l} \partial_j [c_{ijkl}(x)\partial_k u_l(t,x)] = 0.$$

The stiffness tensor can be anisotropic (21 components).

There are three different polarizations, the fastest one of which is quasi-pressure (qP).

Measurements

There are many possible measurements:

- Set any surface displacement and measure the surface force later.
  (Cauchy data: Dirichlet and Neumann boundary values for all solutions of the elastic wave equation.)
- Measure what comes to the surface from unknown interior sources.
- Measure singularities scattering from interfaces or other irregularities.

Measurements

There are many possible measurements:

- Set any surface displacement and measure the surface force later. (Cauchy data: Dirichlet and Neumann boundary values for all solutions of the elastic wave equation.)
- Measure what comes to the surface from unknown interior sources.
- Measure singularities scattering from interfaces or other irregularities.

The goal is to find the material parameters $c_{ijkl}(x)$ and $\rho(x)$.

Measurements

There are many possible measurements:

- Set any surface displacement and measure the surface force later.
  (Cauchy data: Dirichlet and Neumann boundary values for all solutions of the elastic wave equation.)
- Measure what comes to the surface from unknown interior sources.
- Measure singularities scattering from interfaces or other irregularities.

The goal is to find the material parameters $c_{ijkl}(x)$ and $\rho(x)$.

In other words: Can we in principle reliably reconstruct an anisotropic structure inside the Earth from surface measurements?

Geometrization

Step 1: Replace waves with point particles.

$\rightsquigarrow$ Phonons, with or without polarization.

Geometrization

Step 1: Replace waves with point particles.
$\rightsquigarrow$ Phonons, with or without polarization.

Step 2: Encode the material parameters into a such a geometry that the point particles go straight.
$\rightsquigarrow$ Phonons follow qP-Finsler geodesics.

Geometrization

Step 1: Replace waves with point particles.

$\rightsquigarrow$ Phonons, with or without polarization.

Step 2: Encode the material parameters into a such a geometry that the point particles go straight.

$\rightsquigarrow$ Phonons follow qP-Finsler geodesics.

We only use the fastest polarization: It is simple, convex, smooth, and accessible.

Geometrization

Step 1: Replace waves with point particles.
$\rightsquigarrow$ Phonons, with or without polarization.

Step 2: Encode the material parameters into a such a geometry that the point particles go straight.
$\rightsquigarrow$ Phonons follow qP-Finsler geodesics.

We only use the fastest polarization: It is simple, convex, smooth, and accessible.
The arising geometry is not Riemannian but Finsler.

Geometrization

Step 1: Replace waves with point particles.
$\rightsquigarrow$ Phonons, with or without polarization.

Step 2: Encode the material parameters into a such a geometry that the point particles go straight.
$\rightsquigarrow$ Phonons follow qP-Finsler geodesics.

We only use the fastest polarization: It is simple, convex, smooth, and accessible.
The arising geometry is not Riemannian but Finsler.

New problem: Given the boundary data, find the unique Finsler manifold that gives the correct geometric data.

Geometrization

Step 1: Replace waves with point particles.
$\rightsquigarrow$ Phonons, with or without polarization.

Step 2: Encode the material parameters into a such a geometry that the point particles go straight.
$\rightsquigarrow$ Phonons follow qP-Finsler geodesics.

We only use the fastest polarization: It is simple, convex, smooth, and accessible.
The arising geometry is not Riemannian but Finsler.

New problem: Given the boundary data, find the unique Finsler manifold that gives the correct geometric data.

There are theorems about unique determination of Finsler geometries from boundary data.

Degeometrization

We wanted the material parameters c and ρ and we got a Finsler manifold (M, F) with no preferred coordinates.

Degeometrization

We wanted the material parameters c and ρ and we got a Finsler manifold (M, F) with no preferred coordinates.
How is this an answer?

Degeometrization

We wanted the material parameters c and ρ and we got a Finsler manifold (M, F) with no preferred coordinates.
How is this an answer?

The solution has to be degeometrized by fixing the correct coordinates, getting $\rho^{-1}c$ from the qP Finsler geometry, and finally resolving the scalar ρ.

Degeometrization

We wanted the material parameters c and ρ and we got a Finsler manifold (M, F) with no preferred coordinates.
How is this an answer?

The solution has to be degeometrized by fixing the correct coordinates, getting $\rho^{-1}c$ from the qP Finsler geometry, and finally resolving the scalar ρ.

This is work in progress, but much of it seems to be made possible by c being anisotropic.

Degeometrization

We wanted the material parameters c and ρ and we got a Finsler manifold (M, F) with no preferred coordinates.
How is this an answer?

The solution has to be degeometrized by fixing the correct coordinates, getting $\rho^{-1}c$ from the qP Finsler geometry, and finally resolving the scalar ρ.

This is work in progress, but much of it seems to be made possible by c being anisotropic.

Lack of symmetry helps!

Modelling

Modelling includes translating a problem into a new one, solving the new one, and translating the solution back.

Modelling

Modelling includes translating a problem into a new one, solving the new one, and translating the solution back.

This can work. . .

- . . . between physics and mathematics: geophysics $\rightsquigarrow$ inverse boundary value problem
- . . . within mathematics: inverse boundary value problem $\rightsquigarrow$ geometric inverse problem

Modelling

Modelling includes translating a problem into a new one, solving the new one, and translating the solution back.

This can work. . .

- . . . between physics and mathematics: geophysics $\rightsquigarrow$ inverse boundary value problem
- . . . within mathematics: inverse boundary value problem $\rightsquigarrow$ geometric inverse problem

Modelling requires a balanced choice of question. In elastic geometry:

- Riemannian geometry is too narrow to include all the physics.
  (A useful toy model, but only that.)
- Finsler geometry is too general, as some of the key results fail for it.
- The correct model of elastic Finsler geometry is somewhere in between.

Outline

Summary

- The toolbox of inverse problems gives a general framework for indirect measurement problems.
- Wave–particle duality or microlocal analysis: From waves to point particles.
- Fermat’s principle: The particles go straight.
- Balanced modelling both ways makes mathematics meaningful.
- Mathematics and physics benefit from each other, but not automatically.

DISCOVERING MATH at JYU. Since 1863.

Slides and papers available:

http://users.jyu.fi/~jojapeil

Ask for details:

joonas.ilmavirta@jyu.fi