FYSA232 Quantum Mechanics I, part B Fall 2011

Exercise 8 Return to the exercise box in FYS1 lobby before 4 pm on Fri, Nov 18

1. (2 p) Consider the Larmor-precession for spin as discussed in section 5.2.1 (p. 194). First, make sure you understand how the result on p. 195 for the spinor $\psi(t)$ has been obtained, then show that the boxed results on p. 195 hold for the expectation values of the spin components.

2. Let's change the magnetic field in the electron Larmor-precession problem (p. 194) by adding a small x component:

$$\mathbf{B} = B(\mathbf{e_z} + \frac{\sqrt{11}}{5}\mathbf{e_x}).$$

The magnetic moment is still $\hat{\vec{\mu}} = -\mu_B \frac{2}{\hbar}\hat{\mathbf{S}}$ ja $\hat{H} = -\hat{\vec{\mu}} \cdot \mathbf{B}$.

   (a) (1 p) What are the possible energy eigenstates and the corresponding eigenvectors of $\hat{H}$, expressed in the basis $\left|s = \frac{1}{2}, m_s = \pm\frac{1}{2}\right\rangle$?

   (b) (1 p) Let the system be in the state

$$|\psi\rangle = a\,|E_+\rangle + b\,|E_-\rangle\,,$$

   where a ja b are some coefficients and $|E_+\rangle$ and $|E_-\rangle$ are the eigenstates of energy corresponding to higher and lower eigenvalue. What is the spinor describing the state?

3. (a) (1 p) Form the spin-matrices S_x, S_y and S_z (p. 189) of the spin-$\frac{1}{2}$ representation by using the results on p. 186 ($\hat{J}_i = \hat{S}_i$).

   (b) (1 p) Form the spin-matrices S_x, S_y and S_z of the spin-1 representation in a similar way.

   The purpose of this exercise problem is mostly to teach you read these formulae in a correct way. [*Notice:* If you got the correct results from the b part you didn't (surprisingly?) get the matrices Σ_i on pp. 167–168. The reason for this is that the basis $|s, m_s\rangle$ is the so-called spherical basis and one more unitary transformation is required (which exists, see Cronström-Montonen) to transform your results into the generator matrices on pp. 167–168 – thus everything is fine.]

4. (2 p) As on pp. 199–201, carry out the study of the Stern-Gerlach experiment for a <u>spin-1</u> particle. The goal here is to identify the z-aligned momentums of the vector-particle components and to figure out how many distinct groups of points will be formed by the scattered particles on the screen.

5. (2 p) Verify the results on p. 209 for the angular momentum operators $\hat{L}_x$, $\hat{L}_y$, $\hat{L}_z$ and $\hat{L}^2$ in the spherical coordinates. You will need the chain rule of derivatives.

6. (0.5 bonus points) Has something been left unclear so far in the course? Ask something about the course content or related to the course, so that it will be clarified more in the lectures, the exercise sessions, or the course home page.