

Exercise 7 Return to the exercise box in FYS1 lobby before 4 pm on Fri, Nov 11

1. Derive the relation found on the bottom of the p. 167 for the rotations around z -axis and convince yourself with an explicit calculation that $e^{i\alpha\Sigma_z}$ really is identical to the rotation matrix $R_z(\alpha)$ found on p. 167. The generator Σ_z is given on p. 167.
2. Show (s. 174) that angular momentum operator $\hat{L}_i = (\hat{\mathbf{x}} \times \hat{\mathbf{p}})_i = \sum_{j,k} \epsilon_{ijk} \hat{x}_j \hat{p}_k$
 - (a) (1 p) is hermitian.
 - (b) (2 p) satisfies the commutation relations $[\hat{L}_i, \hat{L}_j] = i\hbar \sum_k \epsilon_{ijk} \hat{L}_k$ and $[\hat{L}_i, \hat{\mathbf{L}}^2] = 0$. It is probably easier to do this without the Levi-Civita symbol. For example, you can start by showing that $[\hat{L}_x, \hat{L}_y] = i\hbar \hat{L}_z$ and invoke the permutation symmetry of the indices x, y, z .
3. (a) (1 p) Show that (p. 181) $[\hat{\mathbf{J}}^2, \hat{J}_\pm] = 0$ and $[\hat{J}_z, \hat{J}_\pm] = \pm\hbar \hat{J}_\pm$.
 - (b) (1 p) Analogously to the treatment of the state $\hat{J}_+|j, m\rangle$ on p. 181, show that $\hat{J}_-|j, m\rangle$ is the eigenstate of $\hat{\mathbf{J}}^2$ and \hat{J}_z with eigenvalues corresponding to the quantum numbers j and $m - 1$.
 - (c) (1 p) Show (p. 182), that $\hat{J}_\mp \hat{J}_\pm = \hat{\mathbf{J}}^2 - \hat{J}_z^2 \mp \hbar \hat{J}_z$.
4. (2 p) Show by using the Levi-Civita representation for the angular momentum and the result from the bonus problem below that $[\hat{L}_i, \hat{L}_j] = i\hbar \sum_k \epsilon_{ijk} \hat{L}_k$.
5. (2 bonus points) Show that Levi-Civita symbol satisfies the following identity:

$$\sum_i \epsilon_{ijk} \epsilon_{ist} = \delta_{js} \delta_{kt} - \delta_{jt} \delta_{ks}.$$

If you want, you may use the representation for the Levi-Civita symbol in terms of scalar and vector products of the unit vectors \hat{e}_1, \hat{e}_2 , and \hat{e}_3 : $\epsilon_{ijk} = \hat{e}_i \cdot (\hat{e}_j \times \hat{e}_k)$. In this case first convince yourself that this is indeed so.

6. (0.5 bonus points) Has something been left unclear so far in the course? Ask something about the course content or related to the course, so that it will be clarified more in the lectures, the exercise sessions, or the course home page.