FYSA231 Quantum Mechanics I, part A Fall 2011

Exercise 2 Return to the exercise box in FYS1 lobby before 4 pm on Fri, Sept 23

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1. (4 points) The wave function of a one-dimensional quantum mechanical harmonic oscillator is of the form
$$\psi_0(x) = Ne^{-ax^2},$$
where $N \in \mathbb{C}$ is the normalization constant and $a > 0$.
   (a) Determine the normalization constant N from the requirement $\langle\psi_0|\psi_0\rangle = 1$.
   (b) Calculate the expectation values $\langle x\rangle$, $\langle x^2\rangle$ and the standard deviation Δx.
   (c) What is the most likely value for x?
   (d) Calculate the expectation values $\langle p\rangle$, $\langle p^2\rangle$ and the standard deviation Δp.

   You may use the result $\displaystyle\int_{-\infty}^{\infty} dx\, e^{-bx^2} = \sqrt{\frac{\pi}{b}} \qquad b > 0.$

2. (2p) The ground state wave-function for hydrogen is of the form
$$\psi_0(\vec{x}) = Ne^{-a|\vec{x}|}, \qquad \vec{x} = (x_1, x_2, x_3)$$
where $N \in \mathbb{C}$ is the normalization constant and $a > 0$.
   (a) Determine the normalization constant N from the requirement $\langle\psi_0|\psi_0\rangle = 1$.
   (b) Calculate the expectation values $\langle\vec{x}\rangle$, $\langle\vec{x}^{\,2}\rangle$ and the standard deviation $\Delta\vec{x}$.
   (c) What is the most likely value for the radial distance $r \equiv |\vec{x}|$?

3. (2p) Let's simplify the derivation of the expectation values of the position and momentum operators given on p. 29 of the lecture notes, by reducing the system to a one-dimensional case. Considering thus a one-dimensional system and starting from the definition of the expectation value and using the Schrödinger equation explicitly show that
$$\langle p\rangle = m\frac{d\,\langle x\rangle}{dt}.$$

4. (2p) Starting from the definition of an adjoint operator, show that the nonrelativistic Hamilton operator for three dimensional system
$$\hat{H} = -\frac{\hbar^2\nabla^2}{2m} + V(\hat{\vec{x}})$$
is hermitian, $\hat{H} = \hat{H}^\dagger$.

5. Has something been left unclear so far in the course? Ask something about the course content or related to the course, so that it will be clarified more in the lectures, the exercise sessions, or the course home page. This bonus problem is worth one point (a normal one is worth two), and it is not included in the maximum exercise points.