Exercise 10 Return to the exercise box in FYS1 lobby before 4 pm on Fri, Dec 2

1. Let the unperturbed system be an infinite one-dimensional potential box between $0 \leq x \leq a$ just like we had in the lecture notes on p. 264 (no steps, that is). Let's add a small delta-like perturbation to our system $gV(x) = g\alpha\delta(x - \frac{a}{2})$, where $0 < g \ll 1$ and $|g\alpha| \ll |\frac{\pi^2\hbar^2}{ma}|$.
   (a) (1 p) Find the first-order perturbation theory corrections to the eigenenergies $E_n^{(0)}$ of the system. What happens when n is an even number?
   (b) (1 p) Find the first-order perturbation theory corrections to the wave function describing the ground state $\psi_1^{(0)}(x)$.
   (c) (1 p) Find the second-order perturbation theory corrections to the eigenenergies of this system [Griffiths, Problem 6.4.(a)].

   In parts (b) and (c) you may give your answer as a series. This series can be calculated analytically, but it is not important here.

2. Let the matrix representation of the Hamilton operator of a two-state system be of the form
$$H_0 = \begin{pmatrix} E_0 & 0 \\ 0 & E_0 \end{pmatrix}.$$
   Let's add a general perturbation potential,
$$V = \begin{pmatrix} V_{11} & V_{12} \\ V_{21} & V_{22} \end{pmatrix},$$
   to the system.
   (a) (1 p) Show that the energy corrections according to the first order degenerate perturbation theory are
$$E_{\pm} = \frac{1}{2}[V_{11} + V_{22} \pm \sqrt{(V_{11} - V_{22})^2 + 4|V_{12}|^2}]$$
   (b) (1 p) Show, with as briefly as possible, that the corrected eigenenergies $E_0 + E_{\pm}$ are in this case the exact solutions of the perturbed system! [Tip: It is enough to consider only the determinant equation for computing the eigenvalues of $H = H_0 + V$.]

3. (2 p) Use the variational principle to determine the ground-state energy of the Hydrogen atom. Use the gaussian trial function
$$\varphi(\mathbf{r}) = Ae^{-br^2},$$
   where A is a normalization constant and b the variational parameter. [The result is -11.5 eV which is surprisingly close to the real ground state energy!]

Turn the page!

4. *Griffiths, Example 6.2, pp. 262–264*:
   (a) (1 p) Consider an infinitely high three-dimensional potential box between $0 < x, y, z < a$, (Griffiths [6.30]). Find the eigenenergies E_{n_x,n_y,n_z} and the corresponding wave functions $\psi^0_{n_x,n_y,n_z}(x, y, z)$ of the unperturbed system. What is the energy and its degeneracy of the first excited state? [Tip: First write the wave function in the separated form $\psi(x, y, z) = \psi(x)\phi(y)\gamma(z)$.]

   (b) (2 bonus points) Let's add a perturbation to the system

   $$V(x, y, z) = \begin{cases} V_0, & \text{if} \quad 0 < x < \frac{a}{2} \quad \text{and} \quad 0 < y < \frac{a}{2}; \\ 0, & \text{otherwise.} \end{cases}$$

   Find the first-order perturbation theory corrections to the first excited-state energy.

5. (2 p) *Griffiths example 7.2*: Consider bound states in a one-dimensional delta-potential well

   $$V(x) = -\alpha\delta(x)$$

   (cf. lecture notes pp. 145–146). By using a gaussian trial wave-function as the ground-state wave-function of the system, give an estimate for the ground state energy of the system. Compare your result to the exact result found on p. 146.

6. (0.5 bonus points) Has something been left unclear so far in the course? Ask something about the course content or related to the course, so that it will be clarified more in the lectures, the exercise sessions, or the course home page.