FYSA231 Quantum Mechanics I, part A Fall 2011

Exercise 1 Return to the exercise box in FYS1 lobby before 4 pm on Fri, Sept 16

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1. (a) Show that the complex numbers ψ and ϕ satisfy

$$|\psi+\phi|^2 = |\psi|^2 + |\phi|^2 + 2\Re e(\psi\phi^*) = |\psi|^2 + |\phi|^2 + 2\,|\psi\phi|\cos\left(\alpha_\psi - \alpha_\phi\right),$$

where α_ψ and α_ϕ are the phases of ψ and ϕ.

(b) Starting from the Euler's formula $e^{i\phi} = \cos(\phi) + i\sin(\phi)$, show that

$$\cos(\phi) = \frac{1}{2}\left(e^{i\phi} + e^{-i\phi}\right) \qquad \sin(\phi) = \frac{1}{2i}\left(e^{i\phi} - e^{-i\phi}\right).$$

(c) Calculate the absolute value $|z|$ and the phase $\arg(z)$ of the complex number $z = (1+i)\exp\left(\frac{-i\pi}{2}\right)$.

2. Starting from the definition of an adjoint operator $\hat{A}^\dagger$ on p. 7, show that

(a) $\left\langle \hat{A}^\dagger\psi \middle| \phi \right\rangle = \left\langle \psi \middle| \hat{A}\phi \right\rangle$

(b) $(\hat{A}^\dagger)^\dagger = \hat{A}$

(c) $(\hat{A}\hat{B})^\dagger = \hat{B}^\dagger\hat{A}^\dagger$

3. Show that the eigenvalues of a hermitian operator ($\hat{A} = \hat{A}^\dagger$) are real.

4. Consider a wave function $\psi = \sum_n c_n\psi_n$, where ψ_n is an eigenfunction related to an eigenvalue a_n of a hermitian operator $\hat{A}$. Show that the normalization $\int \mathrm{d}^3x\,\psi^*\psi = 1$ leads to $\sum_n |c_n|^2 = 1$.

5. Show that the definition of an adjoint operator (1D)

$$\int_{-\infty}^{+\infty} dx\,\psi^*\hat{A}^\dagger\phi \equiv \int_{-\infty}^{+\infty} dx\,(\hat{A}\psi)^*\phi,$$

where $\psi \equiv \psi(x)$ and $\phi \equiv \phi(x)$ are normalized wave functions, leads to following equalities

(a) $\left(\frac{d}{dx}\right)^\dagger = -\frac{d}{dx}$

(b) $\left(\frac{d^2}{dx^2}\right)^\dagger = \frac{d^2}{dx^2}$

6. Has something been left unclear so far in the course? Ask something about the course content or related to the course, so that it will be clarified more in the lectures, the exercise sessions, or the course home page. This bonus problem is worth one point (a normal one is worth two), and it is not included in the maximum exercise points.