Interactive Methods in Multiobjective Optimization 1: An Overview

Markus Hartikainen, PhD

Department of Mathematical Information Technology, University of Jyväskylä, Finland
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Interactive Methods

The pseudo code of interactive methods is as follows:

```pseudo
compute a starting solution;
show the solution to the DM;
while the preferred solution has not been found do
    ask for preferences from the DM;
    compute new solution(s) based on the preferences of the DM;
    show the solution(s) to the DM;
end
```

Different interactive methods

- ask for preferences in a different format and
- model the given preferences in a different way
Method of Zionts-Wallenius

- Developed by Zionts and Wallenius in 1976
- It is assumed that linear objective functions are maximized, and that the feasible set $S$ is convex
- Assumes that the DM’s (implicit) preference model is of the weighted sum type
Details of the Zionts-Wallenius Method

- Given a PO solution, the DM is asked to either accept or reject multiple changes which describe adding a unit of different variables that lead to addition in at least one objective.
- The preferences of the decision maker are used to find new constraints to a weighted sum model of the DM’s preferences that model which changes were and were not acceptable.
- The model constructed above is used to find new solutions.
- The constructed model of the DM’s preferences acts as a stopping rule, because one may see that no more improvement on the assumed preference model is possible.
Properties of the Zionts-Wallenius Method

+ Guarantees convergence in a finite number of iterations to the most preferred solution according to the model of the DM’s preferences
+ The demand on the DM is rather low and the questions intuitive
  – The assumptions of the method on the problem are rather restrictive
  – In addition, the assumption on the type DM’s preference model is severe and cannot be verified
The Step Method (STEM)

- Developed by Benayon, de Montgolfier, Tergny and Laritchev in 1971
- Given a PO solution, the DM is asked to choose an objective function where deterioration is allowed in order to improve on the other solutions
- The above is modeled by adding constraints for the objectives that allow the deterioration from the current solution and force the other objectives to be at least as good
- The closest solution to the ideal solution satisfying the added constraints is found (in $L^\infty$-metric)
- The method terminates, when the decision maker so chooses or the feasible set becomes empty
Properties of the STEM Method

- The preference information is intuitive
- The preference model is simple
  - The DM has only a single way to control the search (i.e., choosing the objective to deteriorate)
  - The DM cannot control which objectives are improved
  - The method may converge prematurely into a single solution with poor choices
From Mathematical Convergence to Psychological Convergence

- In early interactive methods, the method provided a stopping criterion.
- The methods used to collect data from the answers of the DM to form a model of the DM’s preferences.
- However, it has been noted that the DM’s are often inconsistent in their answers.
  - Due to learning, changing their minds and mistakes.
- Thus, most more recent interactive methods do not form such a strict model of the DM’s preferences and do not force the DM to stop if he is not happy.
- Psychological convergence refers to the convergence inside DM’s mind.
Pros and Cons of Methods based on Psychological Convergence

+ The DM is allowed to change his mind
– No more guarantee of convergence – convergence depends on the DM
–/+ No stopping criterion
– As there is strict model of the DM’s preferences, it is not easy to say if the DM is stuck
Reference Point Method of Wierzbicki

- Developed by Wierzbicki in 1980
- Based on Wierzbicki’s ideas on how people are not optimizers when making decisions but instead "satisficers"
- The function used for computing new solutions is the achievement scalarizing function given previously
The Procedure of the Reference Point Method

1. The decision maker is asked to provide a reference point $z^{ref}$ (i.e., aspiration levels).
2. The achievement scalarizing function is optimized with the given reference point to produce PO solution $x'$. 
3. For $i = 1, \ldots, k$, the reference point is set as 
   \[ z^{ref, i} = z^{ref} + \| z^{ref} - f(x') \| e^i, \]
   where $e^i$ is the $i^{th}$ unit vector, and the achievement scalarizing function is optimized with those reference points.
4. The $k + 1$ found solutions are presented to the decision maker.
5. If one of the found solutions is satisfactory, then stop. Otherwise, ask for a new reference point and go to step 2.
The idea behind perturbing the reference point is that the decision maker gets a better idea of the possible solutions.

Reference point far away from the PF will provide solutions far away from each other.

Since the decision maker sees always new PO solutions his reference points get closer to the PF.
Properties of the Reference Point Method

+ The preference information is intuitive
+/- The decision maker has a lot of freedom in setting the reference point
  - Requires quite a lot from the decision maker
The Synchronous NIMBUS Method

- Developed by Miettinen and Mäkelä in 1995-2006
- Given a PO solution, the DM is asked to classify the objectives based on desirable changes in those objectives
- Implementations in the IND-NIMBUS® and WWW-NIMBUS include graphical user interfaces
The DM can classify the objectives into classes of

1. objective functions that the decision maker wants to improve as much as possible,
2. wants to improve to a given aspiration level $z_{i}^{asp}$,
3. feels acceptable,
4. allows to get worse until a given bound $\epsilon_{i}$ and
5. allows to change freely for a while

The Synchronous NIMBUS can construct up to four different preference models based on the classification. The number of ways is chosen by the DM.

Produces, thus, up to four PO solutions for each preferences

The DM stops the search whenever he so wishes.
Properties of the Synchronous NIMBUS Method

+ Works basically with any type of problems
+ Uses an intuitive way of giving preference information (classification and bounds/aspirations)
+ Less restrictive as different preference models are available
  – As there are more options for the DM, the needed learning may be more exhaustive
Pareto Navigator

- Assumes minimization of convex objective functions and a convex feasible set $S$
- Assumes a set of precomputed Pareto optimal solutions
- Allows the DM to continuously move on a convex hull of the precomputed Pareto optimal solutions
- Is based on the Pareto Race method by Korhonen in 1988 and, more generally, the reference direction ideas of Korhonen in 1996
Details of the Pareto Navigator Method

- Given a PO solution, the DM can choose aspiration levels for the objectives.
- A ray originating from the current solution on the convex hull and the vector containing the aspiration levels is chosen as the path of the reference point.
- The Achievement Scalarizing Function of Wierzbicki on the convex hull is solved iteratively for multiple points on the ray.
- The DM can stop the progress whenever he wants.
- Once preferred values of objectives on the convex hull are found, the actual PO solution is found again with the Achievement Scalarizing Function.
Illustration of the Pareto Navigator Method

Figure: Figure by Eskelinen et al. in 2010
Properties of the Pareto Navigator

- Especially useful in computationally expensive problems because of the approximation
  - Convexity assumptions are restrictive
- The illusion of continuous movement seems to be intuitive for many DMs
Other Interactive Methods

- Geoffrion-Dyer-Feinberg Method (GDF) in 1972
- Interactive Surrogate Worth Trade-Off Method (ISWT) by Haimes, Hall and Freedman in 1975
- Sequential Proxy Optimization Technique (SPOT) by Sakawa in 1982
- The (Interactive Weighted) Tchebycheff Method by Steuer and Choo in 1983
- Satisficing trade-off method (STOM) by Nakayama and Sawaragi in 1984
- Pareto Race by Korhonen and Wallenius in 1988
- Light Beam Search by Jaskiewicz and Slowinski in 1994
Four types of preference information in interactive methods have been identified by Luque, Ruiz and Miettinen in 2008:

- Trade-off information is supplied e.g., in the Zionts-Wallenius method
- Aspiration levels are set e.g., in Pareto Navigator
- Objectives are classified e.g., in the Synchronous NIMBUS Method
- One or a couple preferable solutions are chosen from a small set of PO solutions in e.g., the Tchebycheff Method

Some methods use a combination of the above preference types
Interactive methods are still an active research field.

Approximation methods are developed to make the computation faster.

New more intuitive ways of providing preferences (including the psychological aspects) are developed.

Graphical user interfaces are developed.

Methods for certain problems (e.g., for planning radiotherapy treatment) are developed.

Multiple methods are combined inside a single framework (e.g., GLIDE, GLocal Interactive Decision Environment, by Luque et al. in 2011).

Also, interactive evolutionary methods are being developed.