FYSH300 fall 2013

Exercise 6, return by Tue Oct 29th at 12.00 to box in the lobby, discussed Tue Oct 29th, at 16.15 in FYS3 **note exceptional time and place**

First midterm exam Fri Nov 15th at 12 covers material up to these exercises and Sect 7 of the lectures. Lectures continue on Oct 28th with K. J. Eskola.

1. Solve the differential equation
$$i\frac{\mathrm{d}a_0(t)}{\mathrm{d}t} = H_{01}e^{-i(E_1-E_0)t} - i\frac{\Gamma}{2}a_0(t), \tag{1}$$
on slide 21 (sect 6) in the lectures, with the initial condition $a_0(0) = 0$. Show that
$$P_0 \equiv |a_0(t \gg \Gamma^{-1})|^2 = \frac{|H_{01}|^2}{(E_1-E_0)^2 + \frac{\Gamma^2}{4}}. \tag{2}$$

2. Examine the processes
$$K^-p \to K^-p \quad \text{and} \quad K^-p \to \overline{K^0}n. \tag{3}$$
Draw the quark diagrams and figure out if a resonance is possible in these processes.

3. Assume that we have the mass matrix
$$H_m = \begin{pmatrix} m+\epsilon & \epsilon & \epsilon \\ \epsilon & m+\epsilon & \epsilon \\ \epsilon & \epsilon & m'+\epsilon \end{pmatrix}. \tag{4}$$
for the $|u\bar{u}\rangle$ $|d\bar{d}\rangle$ $|s\bar{s}\rangle$ spin 1 (vector meson) states. Find out approximately the 3 eigenvectors of the mass matrix, and their eigenvalues, assuming that $\epsilon \ll m, m'$ and $m' > m$. Identify these eigenstates as the ρ^0, ω^0 and ϕ^0 vector mesons, find out the values of m, ϵ, m'. You have to correctly identify which particle corresponds to which eigenvector! Recall that ρ^0 is a member of the isospin triplet $\rho^\pm, \rho^0$ which has strangeness zero.

4. Consider the process $\gamma + p \to \pi^0 + p$. If $E_\gamma = 3K$ (3 Kelvin !), $\sqrt{s} = m_\Delta$ (the Δ resonance mass) and the collision is head-on (incoming p and γ have momenta in the opposite direction), what is the energy of the proton? What are the energies of the incoming and outgoing proton in the CMS? Find out and explain briefly what is the "GZK cutoff".
Bonus points for calculating the full kinematics: Assuming that the scattering angle is $\theta = 0$, calculate the energy of the outgoing proton in the original collision frame. One option could be to figure out the boost velocity between the original and the CMS frames from the initial proton energies and momenta, then apply this boost to the outgoing proton energy in the CMS. Or maybe you can find a more clever way?

5. Look at the explicit expressions for the SU(3) generators t_a from the lecture slides (e.g. slide 29 in section 4). Two of them are diagonal, t_3 and t_8. The unit vectors $\mathbf{e}_1 = (1,0,0)^T$ $\mathbf{e}_2 = (0,1,0)^T$ $\mathbf{e}_3 = (0,0,1)^T$ are eigenvectors of t_3 and t_8. The eigen*value* of t_3 is the 3rd component of isopin I_3 and the eigenvalue of t_8 is $\sqrt{3}/2Y$, where Y is the hypercharge.
   (a) Draw the vectors $\mathbf{e}_i$ as points in the I_3, Y-plane. Assuming that now SU(3) is the SU(3) of quark flavor, identify these points as the u, d, s quarks.
   (b) We define the ladder operators $t_{12\pm} = t_1 \pm it_2$; $t_{45\pm} = t_4 \pm it_5$, $t_{67\pm} = t_6 \pm it_7$. Find out what is the result of when $t_{ab\pm}$ operates on $\mathbf{e}_i$; e.g. $t_{12-}\mathbf{e}_1$. Some of these operations change $\mathbf{e}_i$ into $\mathbf{e}_j$ with $i \neq j$. Draw these as arrows in the I_3, Y-plane figure. What do the other operations do?

6. Now let us consider the SU(3) of color. Show that the totally antisymmetric 3-quark color state is color neutral. In other words, show that operating on the tensor product state
$$|\chi\rangle = \mathbf{e}_1\otimes\mathbf{e}_2\otimes\mathbf{e}_3 + \mathbf{e}_2\otimes\mathbf{e}_3\otimes\mathbf{e}_1 + \mathbf{e}_3\otimes\mathbf{e}_1\otimes\mathbf{e}_2 - \mathbf{e}_1\otimes\mathbf{e}_3\otimes\mathbf{e}_2 - \mathbf{e}_3\otimes\mathbf{e}_2\otimes\mathbf{e}_1 - \mathbf{e}_2\otimes\mathbf{e}_1\otimes\mathbf{e}_3 \tag{5}$$
with the color charge operator
$$t_a^{\text{tot}} \equiv t_a \otimes 1 \otimes 1 + 1 \otimes t_a \otimes +1 \otimes 1 \otimes t_a \tag{6}$$
we get $t_a^{\text{tot}}|\chi\rangle = 0$ for all a. It is enough to check a couple of generators, e.g. $t_3, t_8, t_{45\pm} = t_4 \pm it_5$.