FYSH300 fall 2013

Exercise 5, return by Tue Oct 22nd at 12.00 to box in the lobby, discussed Tue Oct 22nd, at 16.15 in FYS3 **note exceptional time and place**

1. $p\bar{p}$ annihilation happens at rest through the S-wave. Explain using parity why the process $p\bar{p} \to \pi^0\pi^0$ cannot happen through the strong interaction.

2. The $\eta(547)$ meson is a spin-0 particle which decays through the electromagnetic (or strong) interaction to three pions:
$$\eta \to \pi^0 + \pi^0 + \pi^0$$
$$\eta \to \pi^+ + \pi^- + \pi^0.$$
Figure out the parity P_η of the η and explain why the decay processes
$$\eta \to \pi^+ + \pi^-$$
$$\eta \to \pi^0 + \pi^0$$
are not observed.

3. (a) If a meson M decays via the strong interaction to two pions $\pi^+\pi^-$, show that then the relation $P_M = C_M = (-1)^{J_M}$ holds.
   (b) The mesons $\rho^0(770)$ and $f_2^0(1275)$ decay through the strong interaction to a pion pair $\pi^+\pi^-$. The ρ has spin $J_\rho = 1$ and the f_2 has spin $J_{f_2} = 2$. Are the decays $\rho^0 \to \pi^0\gamma$ and $f_2^0 \to \pi^0\gamma$ possible through the electromagnetic interaction? Are the decays $\rho^0 \to \pi^0\pi^0$ and $f_2^0 \to \pi^0\pi^0$ possible through any interaction?

4. The baryonic resonance N^+ has isospin $I = I_3 = \frac{1}{2}$. Show using isospin invariance that
$$\frac{\Gamma(N^+ \to n\pi^+)}{\Gamma(N^+ \to p\pi^0)} = 2. \tag{1}$$

5. Using the isospin invariance of the strong interaction show the following ratio for cross sections
$$\frac{\sigma(pp \to \pi^+ d)}{\sigma(np \to \pi^0 d)} = 2. \tag{2}$$
The deuteron isospin is $I_d = 0$ and the pion isospin is $I_\pi = 1$.

6. Recall the properties of the Pauli matrices $\boldsymbol{\sigma} = (\sigma_1, \sigma_2, \sigma_3)$. The symmetry group of rotations, and of isospin, is SU(2), i.e. the group of unitary 2×2 matrices with determinant=1. SU(2) is a 3-dimensional group, and the 3 *generators* of the spin-1/2 *representation* of the SU(2) group are $\sigma_{1,2,3}/2$. This means that in a rotation by angle $\theta = |\boldsymbol{\theta}|$ around the axis $\boldsymbol{\theta}/\theta$ a spin-1/2 state transforms as
$$\begin{pmatrix} a \\ b \end{pmatrix} \to e^{i\boldsymbol{\theta}\cdot\boldsymbol{\sigma}/2} \begin{pmatrix} a \\ b \end{pmatrix} \tag{3}$$
   (a) Calculate the 2×2 matrix $U(\boldsymbol{\theta}) = e^{i\boldsymbol{\theta}\cdot\boldsymbol{\sigma}/2}$
   (b) Show that this matrix is unitary $U(\boldsymbol{\theta})^\dagger U(\boldsymbol{\theta}) = 1$ and has $\det U(\boldsymbol{\theta}) = 1$
   (c) What is $U(\boldsymbol{\theta})$ with $\boldsymbol{\theta} = (0, 0, \pi)$? What about $(0, \pi, 0)$?