FYSH300 fall 2013

Exercise 2, return by Tue Oct 1st at 14.00 to box in the lobby, discussed Wed Oct 2nd, at 12.15 in FYS5

1. (a) The Lorentz transformation for a four-vector can be written in the form $x' = \Lambda x$, where Λ is a 4×4-matrix. Using the Lorentz boost given on lecture slide 12, write down this matrix for a boost along the z-axis.
   (b) By replacing β by $-\beta$ in the matrix derived above form the matrix for the inverse boost Λ^{-1}, i.e. the matrix for the transformation $x = \Lambda^{-1}x'$.
   (c) Show explicitly using the matrices determined above that $\Lambda^{-1}\Lambda = \mathbf{1}$.
   (d) Show that these matrices satisfy $g\Lambda^T g = \Lambda^{-1}$.

2. Let's consider two consecutive Lorentz boosts in the same direction K $\longrightarrow$ K' $\longrightarrow$ K" as shown on page 15 in the lecture slides.

   (a) Show that the velocity v of the frame K" relative to the frame K is $v = (v_1 + v_2)/(1 + v_1 v_2)$, where v_1 is the velocity of the first boost and v_2 is the velocity of the second boost.
   (b) The rapidity ξ is defined as $v \equiv \tanh \xi$. Show that rapidity is additive in consecutive Lorentz boosts in the same direction. In other words show that the rapidity ξ of the frame K" in the frame K is $\xi = \xi_1 + \xi_2$, where ξ_1 is the rapidity of K' in K and ξ_2 is the rapidity of K" in K'.

3. The rapidity y and pseudorapidity η are defined (lectures, slide 17) as

$$\tanh y \equiv p^z/E \quad \text{and} \quad \tanh \eta \equiv p^z/|\vec{p}|.$$

   The transverse mass m_T and the transverse momentum p_T are defined as

$$m_T \equiv \sqrt{m^2 + p_T^2} \quad \text{and} \quad p_T \equiv \sqrt{p_x^2 + p_y^2} = \sqrt{(p^x)^2 + (p^y)^2}\ .$$

   Now show that

   (a)

$$E = m_T \cosh y \quad \text{and} \quad p^z = m_T \sinh y$$

   (b)

$$|\vec{p}| = p_T \cosh \eta \quad \text{and} \quad p^z = p_T \sinh \eta$$

   (c)

$$\eta \quad = \quad -\ln(\tan \frac{\theta}{2}),$$

   where $\tan \theta \equiv p_T/p^z$.

4. (a) Express the matrix Λ of problem 1 in terms of the rapidity ξ, $v \equiv \tanh \xi$.
   (b) Parametrizing the 4-momentum vector p^μ as $(E, p^1, p^2, p^3) = (m_T \cosh y, p^1, p^2, m_T \sinh y)$ calculate the boosted momentum $\Lambda^\mu{}_\nu p^\nu$ and show that the boost shifts the rapidity y by ξ.

5. Show that the Mandelstam variables s, t and u satisfy the relation

$$s + t + u \quad = \quad m_a^2 + m_b^2 + m_c^2 + m_d^2.$$

6. Derive the results for the energies and momenta of the particles in the TRF and CMS frames expressed in terms of the Mandelstam invariants given on slides 23 and 24 in the lectures. You do not need to do all of them separately, e.g. E_a^*, $|\mathbf{p}_a^*|$, E_a^{TRF} and $|\mathbf{p}_c^{\mathrm{TRF}}|$ are enough.