FYSH300 Particle Physics Autumn 2013

Exercise 11 Return by 14.00 on Tue Dec 3, discussed Wed Dec 4 at 12:15 in FYS5

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1. On page 355 in the lecture notes we defined the helicity operator $\hat{\lambda}$ in its matrix form for a spin-$\frac{1}{2}$-particle.
   (a) Show that for a spin-$\frac{1}{2}$-particle the possible values of helicity are $\pm\frac{1}{2}$.
   (b) Show that the Dirac Hamilton operator $\hat{H}_D = \alpha\cdot\mathbf{p}+\beta m$ and the helicity operator $\hat{\lambda} = \frac{1}{2}\sigma \cdot \frac{\mathbf{p}}{|\mathbf{p}|}$ commute with each other, $[\hat{H}_D, \hat{\lambda}] = 0$. Recall that $(\sigma \cdot \mathbf{p})^2 = \mathbf{p}^2$

2. (a) Show that the Dirac spinors $u_{1,2}(p)$ on p. 259 are eigenspinors of $\hat{\lambda}$ when $\mathbf{p} = |\mathbf{p}|\hat{\mathbf{e}}_z$.
   (b) Using the Dirac-Pauli-representation results for the spinors $u^{(s)}$ on page 259 (let's take $E = E_p > 0$), show that at the ultrarelativistic limit $|\bar{p}| \gg m$ the chirality operator corresponds to the helicity operator i.e. that $\gamma^5 u^{(s)} \cong \hat{\lambda}u^{(s)}$.

3. On page 359 in the lecture notes we defined the projection operators P_L and P_R. Using the properties of γ^5 show that
   (a) $P_L^2 = P_L$
   (b) $P_R^2 = P_R$
   (c) $P_R P_L = P_L P_R = 0$
   (d) $\overline{\psi}_{L,R}\gamma^\mu\psi_{L,R} = \frac{1}{2}\overline{\psi}\gamma^\mu(1 \mp \gamma^5)\psi$

4. **Double points!** Starting from the Lagrangian (14.9) on p. 361, go through in detail the derivation of the Lagrangian (14.21) on p. 365. [One typo in the lecture notes: the eq. on the top of p. 362 should read $\phi \equiv \gamma^\mu\phi_\mu$. You may find more typos, but don't be stuck with these!]