**FYSH555 spring 2014**

Exercise 14, return by Mon 5.5 at 9.15.

1. **Transformation properties of baryon fields:**

   We have seen that the transformation of baryon fields $\Psi$ under chiral rotations is conveniently written as $\Psi \to \Psi' = K(L,R,U)\Psi$ with $L$ and $R$ left- and right-handed rotations and $U$ the Goldstone boson fields (which transform as $U \to U' = RUL^\dagger$). Here, $K(L,R,U)$ is an SU(3) valued function

   $$K(L,R,U) = \sqrt{RUL^\dagger}^{-1} R\sqrt{U}$$

   (a) Verify the property

   $$K(L_1,R_1,R_2UL_2^\dagger)K(L_2,R_2,U) = K((L_1L_2),(R_1,R_2),U)$$

   (b) For a general group element $g=(L,R)$, the transformation behaviour of baryons $\Psi$ depends on the Goldstone Boson fields $U$. What happens in the case of an isospin transformation $R=L=V$? How does $\Psi$ transform under isospin rotations?

2. **Pion nucleon interactions**

   The lowest order chiral Lagrangean involving baryons is in the absence of external fields given by

   $$\mathcal{L}_{\pi N}^{(1)} = \overline{\Psi}\left(i\not{D} - m_N + \frac{g_A}{2}\gamma^\mu\gamma_5 u_\mu\right)\Psi$$

   with the covariant derivative $D_\mu = (\partial_\mu + \Gamma_\mu)$ involving the connection $\Gamma_\mu = \frac{1}{2}[u^\dagger\partial_\mu u + u\partial_\mu u^\dagger]$ and the 'vielbein' $u_\mu = i[u^\dagger\partial_\mu u - u\partial_\mu u^\dagger]$ and $u = \sqrt{U}$ the usual boson fields.

   (a) Expand $u = \exp[i\frac{\vec{\tau}\vec{\phi}}{2F}]$ in powers of $\phi$ and verify that the lowest order interaction terms can be written as

   $$-\frac{1}{2}\frac{g_A}{F}\overline{\Psi}\gamma^\mu\gamma_5\vec{\tau}\partial_\mu\vec{\phi}\Psi - \frac{1}{4F^2}\overline{\Psi}\gamma^\mu\left[(\vec{\tau}\vec{\phi})(\vec{\tau}\partial_\mu\vec{\phi}) - \frac{\partial_\mu\phi\phi + \phi\partial_\mu\phi}{2}\right]\Psi.$$

   (b) Using properties of the Pauli matrices, show that the second term can be simplified to

   $$-\frac{1}{4F^2}\overline{\Psi}\gamma^\mu\epsilon_{abc}\tau^a\phi^b\partial_\mu\phi^c\Psi$$

   (c) Draw the vertices represented by these terms.

3. **Form factor relations**

   As we computed earlier, the divergence of the axial current due to explicit symmetry breaking in SU(2) is given by $\partial_\mu A_i^\mu(x) = m_q P_i(x)$ with $P_i(x)$ the pseudoscalar density.

   (a) Show by inserting both sides of the equation between hadronic states $\langle B|$ and $|A\rangle$ which are eigenstates of the momentum operator with eigenvalues $p_A^\mu$ and $p_B^\mu$ and using translational invariance that

   $$iq_\mu\langle B|A_i^\mu(0)|A\rangle = m_q\langle B|P_i(0)|A\rangle$$

   with $q = p_B - p_A$.

(b) Using Lorentz covariance and isospin symmetry, the matrix element of the axial vector current between nucleon states can be parametrized as

$$\langle N(p')|A_i^\mu(0)|N(p)\rangle = \overline{u}(p')\left[\gamma^\mu G_A(t) + \frac{(p'-p)^\mu}{2m_N}G_P(t)\right]\gamma_5\frac{\tau_i}{2}u(p) \qquad (1)$$

with $G_A(t)$ the axial form factor and $G_P(t)$ the induced pseudoscalar form factor. The matrix element of the pseudoscalar density can be parametrized as

$$m_q\langle N(p')|P_i(0)|N(p)\rangle = \frac{M_\pi^2 F_\pi}{M_\pi^2 - t}G_{\pi N}(t)i\overline{u}(p')\gamma_5\tau_i u(p)$$

with $G_{\pi N}(t)$ the pion-nucleon form factor.
Use these parametrizations to show that

$$2m_N G_A(t) + \frac{t}{2m_N}G_P(t) = 2\frac{M_\pi^2 F_\pi}{M_\pi^2 - t}G_{\pi N}(t)$$

Hint: Use the Dirac equation.