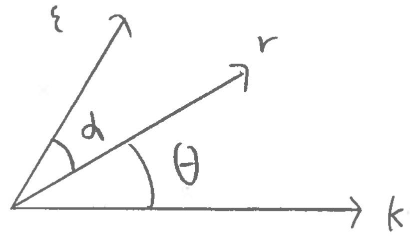
FYSH551 Ex. 5

1.



$$\int d^2\bar{k}\; e^{i\bar{k}\cdot\bar{r}}\; \frac{\bar{\varepsilon}\cdot\vec{k}}{\bar{k}^2}$$

$$\left|\;\begin{aligned}\bar{\varepsilon}\cdot\bar{k} &= \varepsilon k \cos(\alpha+\theta)\\ &= \varepsilon k\left[\cos\alpha\cos\theta - \sin\alpha\sin\theta\right]\\ \bar{k}\cdot\bar{r} &= kr\cos\theta\end{aligned}\right.$$

$$= \int dk\, \cancel{k}\, d\theta\; e^{ikr\cos\theta}\; \frac{\varepsilon \cancel{k}\left[\cos\alpha\cos\theta - \sin\alpha\sin\theta\right]}{\cancel{k^2}}$$

$$= \varepsilon\cos\alpha \int dk\, d\theta\; e^{ikr\cos\theta}\cos\theta - \varepsilon\sin\alpha \underbrace{\int dk\, d\theta\, e^{ikr\cos\theta}\sin\theta}_{\int dk\,\frac{1}{ikr}\Big/_0^{2\pi} e^{ikr\cos\theta} = 0}$$

$$= \varepsilon\cos\alpha \int dk\; 2\pi i\, J_1(kr) \qquad \left|\;\begin{aligned}&\int_0^{2\pi} dx\, \cos x\, e^{ia\cos x}\\ &= 2\pi i\, J_1(a)\end{aligned}\right.$$

$$= 2\pi i\, \varepsilon\cos\alpha\, \frac{1}{r}\underbrace{\int_0^\infty d(kr)\left[-\frac{dJ_0(kr)}{d(kr)}\right]}_{-\Big/_0^\infty J_0(kr) = -(0-1) = 1}$$

$$= 2\pi i\, \varepsilon\cos\alpha\, \frac{1}{r} = \underline{2\pi i\, \frac{\bar{\varepsilon}\cdot\bar{r}}{r^2}}$$

2. BK equation in 0 transverse dimensions:

$$\partial_Y N = \alpha_s N - \alpha_s N^2 \qquad N = N(Y)$$

$$N(Y=0) = N_0 \ll 1$$

Notice fixed point: when $N=1$ $\partial_Y N = 0$

Solve:

$$\frac{dN}{\alpha_s (N-N^2)} = dY$$

$$\int \frac{dN}{N(N-1)} = -\alpha_s \int dY = -\alpha_s \Delta Y \qquad \Big| \frac{1}{N(N-1)} = \frac{1}{N-1} - \frac{1}{N}$$

$$\int \frac{dN}{N-1} - \int \frac{dN}{N} = -\alpha_s \Delta Y$$

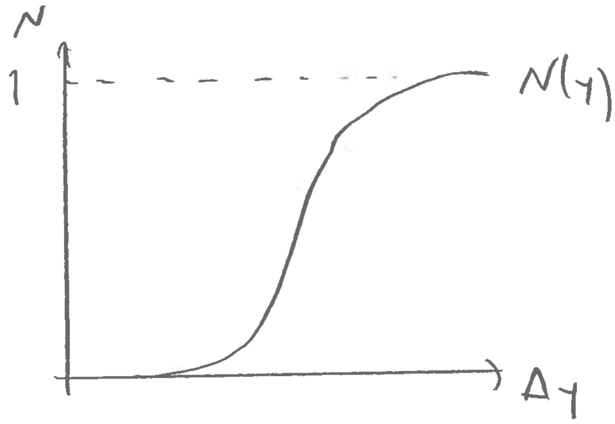
$$\ln |N-1| - \ln |N| = -\alpha_s \Delta Y + C$$

$$\frac{N-1}{N} = A e^{-\alpha_s \Delta Y} \qquad (A = e^C)$$

$$N\left(1 - A e^{-\alpha_s \Delta Y}\right) = 1$$

$$N = \frac{e^{\alpha_s \Delta Y}}{e^{\alpha_s \Delta Y} - A}, \quad \text{where } N_0 = \frac{1}{1-A}$$

$$A = \frac{N_0 - 1}{N_0} < 0$$


N
1
N(y)
Δy

3. 103 citations in ~3 years!

In the paper a running coupling BK equation is used, in the lectures we derived the BK at fixed α_s,

$$K(r_1, r_2, r) = \frac{\alpha_s N_c}{2\pi^2} \frac{r^2}{r_1^2 r_2^2}$$

In forward rapidity a particle is produced in the proton/deuteron fragmentation region (in p+A collision)

→ need a lot of longitudinal momentum from the proton (large x) and very little long. momentum from the nucleus (small x $\hat{=}$ CGC).

Eq. (1) is "hybrid formalism": large-x proton is described by PDF (xf ~ propability to find a quark), scattering off the small-x nucleus is described by the dipole amplitude N. = CGC

$$dN \sim \int \frac{dz}{z^2} \; x_1 f_q(x_1, \mu) \, N\left(x_2, \frac{p_T}{z}\right) D_h(z, \mu)$$

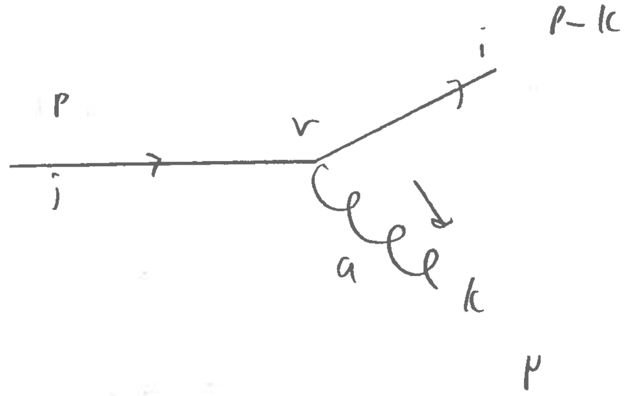
take large-x quark/gluon from proton

quark/gluon scatters off the nucleus = CGC

scattered parton fragments to a measured particle

At midrapidity we would have $x_1 \sim x_2$ and this factorization would not work.

4.



$$A_\mu^a = -ig\, t^a \frac{-ig_{\mu\nu}}{k^2+i\varepsilon} \underbrace{\bar{u}_\sigma(p-k)\,\gamma^\nu\, u_\sigma(p)}_{\text{eikonal: } 2p^\nu} (2\pi)\,\delta((p-k)^2)$$

$$= 2g\,t^a \frac{p_\mu}{k^2+i\varepsilon}\, 2\pi\,\delta((p-k)^2)$$

$\Rightarrow$ only A^+ component!

$$A_+^a(x) = 2g\,t^a \int \frac{d^4k}{(2\pi)^4} e^{-ik\cdot x} \frac{p_+}{k^2+i\varepsilon}\, 2\pi \underbrace{\delta((p-k)^2)}_{\delta(k^2-2p^+k^-)}$$

$$A^{+a}(x) = 2g\,t^a \int \frac{d^4k}{(2\pi)^4} e^{-ik^+x^- - ik^-x^+ + i\underset{\sim}{k}\cdot\underset{\sim}{x}} \frac{p^+}{k^2+i\varepsilon}$$

$$\frac{1}{2p^+}\,\delta\left(k^- - \frac{k^2}{2p^+}\right) 2\pi$$

Integrate over k^-:

$$= 2 g\, t^a \int \frac{d^3k}{(2\pi)^3}\ \frac{1}{2}\ \frac{1}{k^2+i\epsilon}\ e^{-ix^-k^+ - i x^+ \underbrace{\frac{k^2}{2p^+}}_{\to 0} + i \underset{\sim}{k}\cdot\hat{x}}$$

$$\left(\text{and } k^2 = k^+\frac{k^2}{2p^+} - \underset{\sim}{k}^2 \approx -\underset{\sim}{k}^2\right)$$

$$d^3k = dk^+ d\underset{\sim}{k}$$

$$= g\, t^a \int \frac{d^2\underset{\sim}{k}}{(2\pi)^3}\ \underbrace{dk^+ e^{-ix^-k^+}}_{2\pi\,\delta(x^-)}\ \frac{e^{i\underset{\sim}{k}\cdot\hat{x}}}{\underset{\sim}{k}^2+i\epsilon}$$

$$= g\, t^a \delta(x^-) \underbrace{\int \frac{d^2\underset{\sim}{k}}{(2\pi)^2}\ \frac{e^{i\underset{\sim}{k}\cdot\hat{x}}}{\underset{\sim}{k}^2+i\epsilon}}_{\frac{1}{(2\pi)^2}\int dk\, k\, d\theta\ \frac{e^{ikx\cos\theta}}{k^2+i\epsilon}}$$

$$= \frac{1}{2\pi}\underbrace{\int dk\ \frac{k J_0(kx)}{k^2+i\epsilon}}_{\int_\Lambda^\infty \frac{kJ_0(kx)}{k^2} = -\ln(x\Lambda) + \text{const} + O(\Lambda)}$$

$$= -\frac{g}{2\pi}\, t^a\, \delta(x^-)\, \ln(x\Lambda)$$

5.

$$F^{\mu\nu} = \partial^\mu A^\nu - \partial^\nu A^\mu + ig[A^\mu, A^\nu]$$

$$A_i = A_{i,a} t^a = -\frac{i}{g} U(x) \partial_i U^\dagger(x)$$

$U = U(x)$

$$igF^{ij} = (\partial^i U)\partial^j U^\dagger + \underbrace{U \partial^i \partial^j U^\dagger}$$

$$- (\partial^j U)\partial^i U^\dagger - \underbrace{U \partial^j \partial^i U^\dagger}$$

$$+ \left[U(\partial^i U^\dagger) U \partial^j U^\dagger - U(\partial^j U^\dagger) U \partial^i U \right]$$

Noticing $UU^\dagger = 1 \Rightarrow (\partial^i U)U^\dagger + U\partial^i U^\dagger = 0$

$$U \partial^i U^\dagger = -(\partial^i U)U^\dagger$$

we get

$$igF^{ij} = (\partial^i U)\partial^j U^\dagger - (\partial^j U)\partial^i U^\dagger$$

$$+ \left[-(\partial^i U)\underbrace{U^\dagger U}_{1} \partial^j U^\dagger + (\partial^j U)\underbrace{U^\dagger U}_{1} \partial^i U \right]$$

$= 0 \Rightarrow$ no long. (color) magnetic field.

write $A^2_i = -\frac{i}{g} V(x)\, \partial_i V^+(x)$

Now for $A = A^1 + A^2$

$$F^{ij} = \partial^i (A^1 + A^2)^j - \partial^j (A^1 + A^2)^i + ig\left[(A^1+A^2)^i, (A^1+A^2)^j\right]$$

$$\left.\begin{aligned} &= \partial^i A^{1,j} - \partial^j A^{1,i} + ig\left[A^{1,i}, A^{1,j}\right] \\ &\quad + \partial^i A^{2,j} - \partial^j A^{2,i} + ig\left[A^{2,i}, A^{2,j}\right] \end{aligned}\right\} = 0$$

$$+ \underbrace{ig\left[A^{1,i}, A^{2,j}\right] + ig\left[A^{2,i}, A^{1,j}\right]}$$

or, in terms of U, V:

$$\Rightarrow ig F^{ij} = U(\partial^i U^+) V \partial^j V^+ - V(\partial^j V^+) U \partial^i U^+$$

$$+ V(\partial^i V^+) U \partial^j U - U(\partial^i U^+) V \partial^j V$$