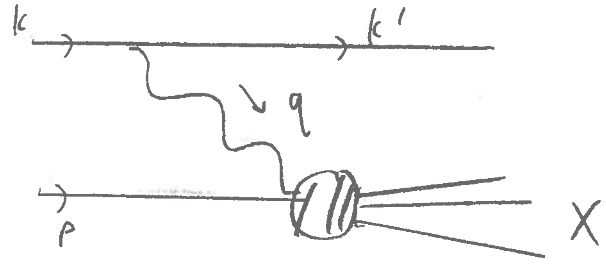
FYSH551 EX. 4

1. 

DIS event

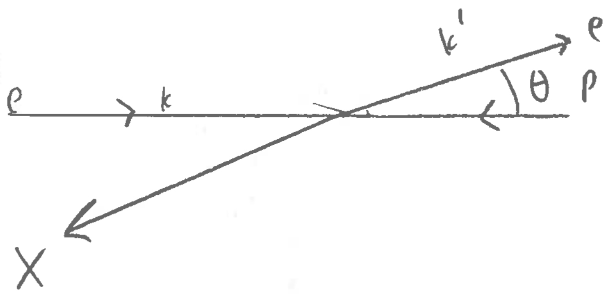
$E_e = 27.5\ \mathrm{GeV}$

$E_p = 920\ \mathrm{GeV}$

$k = (E_e, \underset{\sim}{0}, E_e)$,

$p = (E_p, \underset{\sim}{0}, -E_p)$

Lab frame



$q = k - k'$

$Q^2 = -q^2$

$x = \dfrac{Q^2}{Q^2 + W^2}$, $W^2 = (p+q)^2$

$$Q^2 = 2\,k \cdot k' = 2\,E E' (1 - \cos\theta)$$

$$W^2 = q^2 + 2p\cdot q = -Q^2 + 2\,p\cdot(k - k')$$

$$= -Q^2 + 2\left(E_p E_e - E_p E_e \cos\pi\right) - 2\left(E_p E_e' - E_p E_e' \underbrace{\cos(\pi-\theta)}_{-\cos\theta}\right)$$

Two equations with two unknowns E_e' and $\cos\theta$

1. $Q^2 = 50\ \mathrm{GeV}^2$, $x = 0.1$

$\Rightarrow$ $E_e' = 27.8\ \mathrm{GeV}$

$\theta = 14.69°$

$Q^2 = 3\ \mathrm{GeV}^2$, $x = 0.001$

$E_e' = 26.71\ \mathrm{GeV}$

$\theta = 3.66°$

2. $$\frac{d\sigma}{dE'\,d\Omega} = \frac{\alpha_{em}^2}{2mQ^2}\,\frac{E'}{E}\,L_{\mu\nu}W^{\mu\nu}$$

$$L_{\mu\nu} = 2\left(k_\mu k'_\nu + k'_\mu k_\nu - g_{\mu\nu}\, k\cdot k'\right)$$

$$W_{\mu\nu} = -2\left(g_{\mu\nu} + \frac{q_\mu q_\nu}{Q^2}\right) F_1(x,Q^2)$$

$$+ \frac{2}{p\cdot q}\left[\left(p_\mu + \frac{p\cdot q}{Q^2} q_\mu\right)\left(p_\nu + \frac{p\cdot q}{Q^2} q_\nu\right)\right] F_2(x,Q^2)$$

1. $$L_{\mu\nu}\cdot\left[-2\left(g_{\mu\nu} + \frac{q_\mu q_\nu}{Q^2}\right)\right] = -4\Big(k\cdot k' + k\cdot k' - \overbrace{g_{\mu\nu}\, g^{\mu\nu}}^{4}\, k\cdot k'$$

$$+ \frac{1}{Q^2}\left(q\cdot k\; q\cdot k' + q\cdot k'\; q\cdot k - q^2\, k\cdot k'\right)\Big)$$

$$= -4\left(-2\,k\cdot k' + 2\,\frac{q\cdot k\; q\cdot k'}{Q^2} + k\cdot k'\right)$$

$$= -4\left(-k\cdot k' + 2\,\frac{q\cdot k\; q\cdot k'}{Q^2}\right)$$

Notice: $Q^2 = -(k-k')^2 = 2k\cdot k' \quad \Rightarrow k\cdot k' = \frac{Q^2}{2}$

$q\cdot k = (k-k')\cdot k = -k\cdot k' = -\frac{Q^2}{2} = -q\cdot k'$

$$\Rightarrow \quad = -4\left(-\frac{Q^2}{2} - 2\,\frac{\frac{1}{4}Q^2Q^2}{Q^2}\right) = +4Q^2$$

Thus the coefficient of F_1 in $L_{\mu\nu}W^{\mu\nu}$ is

$$+4Q^2$$

2nd part:

$$2\left(k_\mu k'_\nu + k_\nu k'_\mu - g_{\mu\nu}\overbrace{k\cdot k'}^{Q^2/2}\right)\underbrace{\frac{2}{P\cdot q}}_{\frac{4x}{Q^2}}\left[\left(p^\mu + \underbrace{\frac{P\cdot q}{Q^2}}_{\frac{1}{2x}} q^\mu\right)\left(p^\nu + \frac{P\cdot q}{Q^2}q^\nu\right)\right]$$

Use: $k\cdot q = \frac{-Q^2}{2}$, $k'\cdot q = \frac{Q^2}{2}$, $P\cdot k = \frac{P\cdot q}{y}$

$P\cdot k' = P\cdot(k-q) = \frac{P\cdot q}{y} - P\cdot q = \frac{Q^2}{2yx} - \frac{Q^2}{2x}$

$P\cdot P = m^2$

$$\Rightarrow \quad -4m^2 x + \frac{4Q^2}{xy^2} - \frac{4Q^2}{xy}$$

$$\therefore L_{\mu\nu}W^{\mu\nu} = 4Q^2 F_1(x,Q^2)$$
$$+ 4\left(\frac{Q^2}{xy^2} - \frac{Q^2}{xy} - m^2 x\right)F_2(x,Q^2)$$

3. TRF: $p^\mu = (m, \vec{0})$ $\quad q^\mu = (\nu, \underset{\sim}{Q}, \sqrt{Q^2+\nu^2})$ $\quad p \cdot q = m\nu$

$$d^{L,00} = \frac{Q^2}{m^2(\nu^2+Q^2)} \left(m + \frac{m\nu}{Q^2}\nu\right)\left(m + \frac{m\nu}{Q^2}\nu\right)$$

$$= \frac{Q^2}{(\nu^2+Q^2)} \left(1 + \frac{\nu^2}{Q^2}\right)^2$$

$$d^L_{10} = d^L_{20} = d^L_{01} = d^L_{02} = d^L_{11} = d^L_{22} = 0$$

$$d^{L,03} = \frac{Q^2}{m^2(\nu^2+Q^2)} = \left(m + \frac{m\nu}{Q^2}\nu\right)\left(0 + \frac{m\nu}{Q^2}\sqrt{Q^2+\nu^2}\right)$$

$$= \frac{Q^2}{\nu^2+Q^2}\left(1 + \frac{\nu^2}{Q^2}\right)\frac{\nu}{Q^2}\sqrt{Q^2+\nu^2}$$

$$d^{L30} = d^{L03}$$

$$d^{L,33} = \frac{Q^2}{m^2(\nu^2+Q^2)} \frac{m\nu}{Q^2}\sqrt{Q^2+\nu^2}\,\frac{m\nu}{Q^2}\sqrt{Q^2+\nu^2} = \frac{\nu^2}{Q^2}$$

$$\therefore\ d^{L,\mu\nu} = \underbrace{\frac{Q^2}{\nu^2+Q^2}\left(1 + \frac{\nu^2}{Q^2}\right)}_{1} \begin{pmatrix} 1+\frac{\nu^2}{Q^2} & 0 & 0 & \frac{\nu}{Q^2}\sqrt{Q^2+\nu^2} \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ \frac{\nu}{Q^2}\sqrt{Q^2+\nu^2} & 0 & 0 & \frac{\nu^2}{Q^2} \end{pmatrix}$$

$$d^{00} = -\left(1 + \frac{v^2}{Q^2}\right) \qquad d_{11} = d_{22} = 1$$

$$d_{33} = 1 - \frac{Q^2+v^2}{Q^2} = -\frac{v^2}{Q^2} \qquad d_{30} = -\frac{v\sqrt{Q^2+v^2}}{Q^2} = d_{03}$$

$$d^{\mu\nu\, T} = \begin{pmatrix} -\left(1+\frac{v^2}{Q^2}\right) & 0 & 0 & -\frac{v}{Q^2}\sqrt{Q^2+v^2} \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ -\frac{v}{Q^2}\sqrt{Q^2+v^2} & 0 & 0 & -\frac{v^2}{Q^2} \end{pmatrix}$$

Thus

$$2\, d^{\mu\nu}_{T} = \begin{pmatrix} 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix}$$

3. cont

Brick frame : $p^\mu = \left(m\sqrt{1+v^2/Q^2},\ \underset{\sim}{0},\ \frac{-mv}{Q} \right)$

$q^\mu = (0, \underset{\sim}{0}, Q)$

$p \cdot q = mv$

$$d^{L\,00} = \frac{Q^2}{m^2(v^2+Q^2)} \, m^2\left(1+\frac{v^2}{Q^2}\right) = 1$$

$$d^{L\,10} = d^{L\,20} = d^{L\,01} = d^{L\,02} = 0 = d^{L\,11} = d^{L\,22}$$

$$d^{L\,33} = \frac{Q^2}{m^2(v^2+Q^2)} \left(\frac{-mv}{Q} + \frac{mv}{Q^2} Q \right)\left(\frac{-mv}{Q} + \frac{mv}{Q^2} Q \right) = 0$$

$$d^{L\,13} = d^{L\,23} = d^{L\,32} = d^{L\,31} = 0$$

$$d^{L\,03} = \frac{Q^2}{m^2(v^2+Q^2)} \left(m\sqrt{1+\frac{v^2}{Q^2}} + 0 \right)\left(\frac{-mv}{Q} + \frac{mv}{Q^2} Q \right)$$

$$= 0 = d^{L\,30}$$

$$\Rightarrow \quad d^L = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix}$$

$$d^{00} = -(1+0) = -1$$

$$d^{ij} = 0 \qquad \text{when} \qquad i \neq j = 1,2,3$$

$$d^{33} = -\left(-1 + \frac{Q^2}{Q^2}\right) = 0$$

$$d^{11} = -(-1) = 1 = d^{22}$$

$$d = \begin{pmatrix} -1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix}$$

$$\Rightarrow \quad d^T = \frac{1}{2} \begin{pmatrix} 0 & & & 0 \\ & 1 & & \\ & & 1 & \\ 0 & & & 0 \end{pmatrix}$$

4.

$$\sigma^{\gamma^* p}_{T,L} = \frac{4\pi^2 \alpha_{em}}{Q^2(1-x)} \times d^{T,L}_{\mu\nu} W^{\mu\nu}$$

$$d^L_{\mu\nu} W^{\mu\nu} = \frac{Q^2}{m^2(\nu^2+Q^2)} \left(P_\mu + \overbrace{\frac{p\cdot q}{Q^2}}^{1/2x} q_\mu \right)\left(P_\nu + \frac{p\cdot q}{Q^2} q_\nu \right)$$

$$\times \left[-2\left(g^{\mu\nu} + \frac{q^\mu q^\nu}{Q^2} \right) F_1 + \frac{2}{p\cdot q} \left[\left(p^\mu + \frac{p\cdot q}{Q^2} q^\mu \right)\left(p^\nu + \frac{p\cdot q}{Q^2} q^\nu \right.\right.\right.$$

substituting the invariants as in prob. 2 and noticing that $\nu^2 = \frac{p\cdot q}{m_N} = \frac{Q^2}{2x m_N}$

we get directly (after some algebra / FeynCalc)

$$d^L_{\mu\nu} W^{\mu\nu} = -2F_1 + \left(\frac{4m^2 x}{Q^2} + \frac{1}{x} \right) F_2$$

Similarly $d^T_{\mu\nu} W^{\mu\nu} = \frac{1}{2}\left(d_{\mu\nu} + d^L_{\mu\nu} \right) W^{\mu\nu}$

$= (\text{simplify}) = \quad 2F_1$

$$\therefore \quad \sigma_L^{\gamma^* p} = \frac{4\pi^2 \alpha_{em}}{Q^2(1-x)} \left(-2x\, F_1(x, Q^2) + \left(\frac{4m^2x^2}{Q^2} + 1 \right) F_2(x, Q^2) \right)$$

and

$$\sigma_T^{\gamma^* p} = \frac{4\pi^2 \alpha_{em}}{Q^2(1-x)} \, 2x\, F_1(x, Q^2)$$

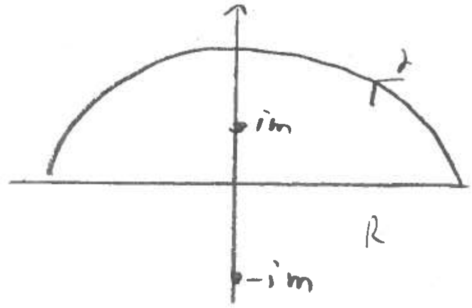
Now

$$\frac{Q^2(1-x)}{4\pi^2 \alpha_{em}} \frac{Q^2}{Q^2+4m^2x^2} (\sigma_T + \sigma_L)$$

$$= \frac{Q^2}{Q^2+4m^2x^2} \left(\frac{4m^2x^2}{Q^2} + 1 \right) F_2 = F_2$$

Same result as in the ZEUS paper.

5. $\int_{-\infty}^{\infty} dk \, \frac{e^{ikx}}{k^2+m^2}$



$$= \int_{-\infty}^{\infty} dk \, \frac{e^{ikx}}{(k+im)(k-im)}$$

$$= 2\pi i \operatorname*{Res}_{k\to im} \frac{e^{ikx}}{(k+im)(k-im)} = 2\pi i \, \frac{e^{i(im)x}}{im+im} = \frac{e^{-mx}}{m} \pi$$

- As the integral along the curve on the upper half plane vanishes:

$$\left| \int_\gamma dz \, \frac{e^{izx}}{z^2+m^2} \right| \leq \int_0^{\pi} d\theta \left| Re^{i\theta} \frac{\exp(ixRe^{i\theta})}{R^2 e^{i2\theta}+m^2} \right|$$

$$= \int_0^{\pi} d\theta \, \frac{e^{-Rx\sin\theta}}{|Re^{2i\theta}+m^2/R|} \to 0 \quad \text{as } R\to\infty$$

$$\int d^2\underset{\sim}{k}\ \frac{e^{i\underset{\sim}{k}\cdot\underset{\sim}{r}}}{\underset{\sim}{k}^2+m^2} = \int_0^\infty dk \underbrace{\int_0^{2\pi} d\theta\ k\ e^{ikr\cos\theta}}_{k\,2\pi J_0(kr)}\ \frac{1}{k^2+m^2} \qquad \Big|\ k=|\underset{\sim}{k}|$$

$$= 2\pi\int_0^\infty dk\ J_0(kr)\ \frac{k}{k^2+m^2}$$

$$= 2\pi K_0(mr)$$