FYSH555 EX. 12

1. $V = \frac{m^2}{2}\phi_i\phi_i + \frac{\lambda}{4}(\phi_i\phi_i)^2 + a\,\phi_2$

minimum at $\bar\phi_1 = 0$, $\bar\phi_2 = -\sqrt{\frac{-m^2}{\lambda}} + \frac{a}{2m^2}$,

$m < 0$

a) Masses for the physical fields are the coefficients of the (field)^2 terms.

Write $\phi_2 = \bar\phi_2 + \sigma$

For ϕ_1:

$$\frac{1}{2}m_1^2\phi_1^2 = \frac{m^2}{2}\phi_1\phi_1 + \frac{\lambda}{4}2\phi_1\phi_1\bar\phi_2\bar\phi_2$$

note: no $\bar\phi_2 + \sigma$, as $\sim \phi_1\phi_1\sigma\sigma$ and $\phi_1\phi_1\sigma\bar\phi_2$ are interaction terms

$$m_1^2 = m^2 + \frac{\lambda}{2}\bar\phi_2^2$$

$$= m^2 - m^2 - 2\lambda\frac{a}{2m^2}\sqrt{\frac{-m^2}{\lambda}} + O(a^2)$$

$$m_1^2 = a\sqrt{\frac{\lambda}{-m^2}} \rightarrow 0 \text{ when } a \rightarrow 0$$

For σ we collect terms $\sim \sigma^2$ and no interaction! from

$$\frac{m^2}{2}(\bar\phi_2+\sigma)(\bar\phi_2+\sigma) + \frac{\lambda}{4}(\bar\phi_2+\sigma)^4$$

$$\Rightarrow \frac{1}{2}m_\sigma^2\sigma^2 = \frac{m^2}{2}\sigma^2 + \frac{\lambda}{4}\left(6\bar\phi_2^2\sigma^2\right)$$

$$= \frac{1}{2}m^2\sigma^2 + \frac{3}{2}\lambda\left(\frac{-m^2}{\lambda} - \frac{a}{m^2}\sqrt{\frac{-m^2}{\lambda}}\right)\sigma^2 + O(a^2)$$

$$m_\sigma^2 = m^2 - 3m^2 + 3a\sqrt{\frac{\lambda}{-m^2}}$$

$$m_\sigma^2 = -2m^2 + 3a\sqrt{\frac{\lambda}{-m^2}} \xrightarrow{a \to 0} -2m^2$$

b) 1 massive particle σ

1 (in generic case $n-1$) massless Goldstone bosons.

With explicit symmetry breaking the Goldstone bosons acquire mass ~ symmetry breaking parameter a.

2. $$\lambda_a \phi_a = \begin{pmatrix} \phi_3 + \frac{1}{\sqrt{3}}\phi_8 & \phi_1 - i\phi_2 & \phi_4 - i\phi_5 \\ \phi_1 + i\phi_2 & -\phi_3 + \frac{1}{\sqrt{3}}\phi_8 & \phi_6 - i\phi_7 \\ \phi_4 + i\phi_5 & \phi_6 + i\phi_7 & -\frac{2}{\sqrt{3}}\phi_8 \end{pmatrix},$$

as $\lambda_1 = \begin{pmatrix} 0 & 1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}$, $\lambda_2 = \begin{pmatrix} 0 & -i & 0 \\ i & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}$, ..., $\lambda_8 = \frac{1}{\sqrt{3}}\begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & -2 \end{pmatrix}$

are the Gell-Mann matrices.

From the lectures

$$\lambda_a \phi_a = \begin{pmatrix} \pi^0 + \frac{1}{\sqrt{3}}\eta & \sqrt{2}\pi^+ & \sqrt{2}K^+ \\ \sqrt{2}\pi^- & -\pi^0 + \frac{1}{\sqrt{3}}\eta & \sqrt{2}\bar{K}^0 \\ \sqrt{2}K^- & \sqrt{2}K^0 & -\frac{2}{\sqrt{3}}\eta \end{pmatrix}$$

$\Rightarrow$ solve $\pi^0, \pi^\pm, K^0, K^\pm, \eta, \bar{K}^0$ as a function of ϕ_i.

$\eta = \phi_8$

$\pi^0 = \phi_3$

$\pi^- = \frac{1}{\sqrt{2}}(\phi_1 + i\phi_2)$

$\pi^+ = \frac{1}{\sqrt{2}}(\phi_1 - i\phi_2)$

$K^0 = \frac{1}{\sqrt{2}}(\phi_6 + i\phi_7)$

$\bar{K}^0 = \frac{1}{\sqrt{2}}(\phi_6 - i\phi_7)$

$K^+ = \frac{1}{\sqrt{2}}(\phi_4 - i\phi_5)$

$K^- = \frac{1}{\sqrt{2}}(\phi_4 + i\phi_5)$

3. $\mathcal{L}_m = -\bar{q}_R M q_L - \bar{q}_L M^\dagger q_R$

$M \to R M L^\dagger = M'$, $\quad q_R \to R q_R = q_R'$ $\quad q_L \to L q_L = q_L'$

$\mathcal{L}_m \to \mathcal{L}_m' = -\bar{q}_R' M' q_L' - \bar{q}_L' M'^\dagger q_R$

$= -(R q_R)^\dagger \gamma^0 R M L^\dagger L q_L - (L q_L)^\dagger \gamma^0 (R M L^\dagger)^\dagger R q_R$

$= -q_R^\dagger \gamma^0 R^\dagger R M L^\dagger L q_L - q_L^\dagger \gamma^0 L M R^\dagger R q_R$

$= -\bar{q}_R M q_L - \bar{q}_L M q_R$

b) $U \to R U L^\dagger$

$\mathcal{L}_{SB} = \frac{F_0^2 B_0}{2} \mathrm{Tr}(M U^\dagger + U M^\dagger)$

$\mathrm{Tr}(M U^\dagger + U M^\dagger) \to \mathrm{Tr}(M' U'^\dagger + U' M'^\dagger)$

$= \mathrm{Tr}(R M L^\dagger (R U L^\dagger)^\dagger + R U L^\dagger (R M L^\dagger)^\dagger)$

$= \mathrm{Tr}(R M L^\dagger L U^\dagger R^\dagger + R U L^\dagger L M^\dagger R^\dagger)$

$= \mathrm{Tr}[R (M U^\dagger + U M^\dagger) R^\dagger]$

$= \mathrm{Tr}(M U^\dagger + U M^\dagger)$

$\Rightarrow \mathcal{L}_{SB}$ is invariant.

c) Parity transformation

$$\phi(t,\bar{x}) \xrightarrow{P} -\phi(t,-\bar{x})$$

$$U(t,\bar{x}) = \exp\left(i \frac{\lambda_a \phi_a(t,\bar{x})}{F_0}\right) \xrightarrow{P} \exp\left(-i \frac{\lambda_a \phi_a(t,-\bar{x})}{F_0}\right)$$

$$= U^\dagger(t,-\bar{x}) \text{ as } \lambda_a^\dagger = \lambda_a.$$

Using also the fact $M = M^\dagger$ we get

$$\mathrm{Tr}[M U^\dagger(t,\bar{x}) - U(t,\bar{x}) M^\dagger]$$

$$\xrightarrow{P} \mathrm{Tr}[\underbrace{M}_{=M^\dagger} U(t,-\bar{x}) - U^\dagger(t,-\bar{x}) \underbrace{M^\dagger}_{=M}]$$

$$= \mathrm{Tr}[M^\dagger U(t,-\bar{x})] - \mathrm{Tr}[U^\dagger(t,-\bar{x}) M]$$

$$= \mathrm{Tr}[U(t,-\bar{x}) M^\dagger] - \mathrm{Tr}[M U^\dagger(t,-\bar{x})]$$

$$= -\mathrm{Tr}[M U^\dagger(t,-\bar{x}) - U(t,-\bar{x}) M^\dagger].$$