FYSH555 Ex. 11

1. $\mathcal{L} = g \bar{N} \gamma_5 \bar{T} \bar{N} \cdot \bar{\phi}$

a)

$\bar{\phi} = (\phi_1, \phi_2, \phi_3)^T$

$\bar{N} = (p, n)^T$

(note: p, n are Dirac spinors)

$T_1 = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}$ $T_2 = \begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix}$ $T_3 = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}$

$T_1 N = \begin{pmatrix} n \\ p \end{pmatrix}$ $T_2 N = \begin{pmatrix} -in \\ ip \end{pmatrix}$ $T_3 N = \begin{pmatrix} p \\ -n \end{pmatrix}$

$\bar{N}^T \gamma_5 \bar{T} N \cdot \phi$: has terms that contain the following structures:

$\underbrace{p n \phi_1 \quad n p \phi_1}_{\text{from } T_1 N}$, $-i\, p n \phi_2$, $i\, n p \phi_2$, $p p \phi_3$, $-n n \phi_3$

b) Clearly ϕ_3 is neutral (as we have e.g. $p \to p \phi_3$)
=> identify $\phi_3 \equiv \pi^0$

As we ~~can~~ must have $p \to n \pi^+$ (charge conservation),
and all reactions of the form $p \to n X$ from above are
$\sim p\, n \phi_1$ and $\sim -i p n \phi_2$ => $\pi^+ = \phi_1 - i \phi_2$

Similarly we have $n \to p \pi^-$ which comes from $\sim n p \phi_1$ and $\sim i n p \phi^2$

$$\Rightarrow \underline{\pi^- = \phi_1 + i\phi_2}$$

Then $\bar{\phi} = \tau_i \phi_i = \begin{pmatrix} \pi^0 & \sqrt{2}\pi^+ \\ \sqrt{2}\pi^- & -\pi^0 \end{pmatrix}$.

2. a) $P_R = \frac{1}{2}(1+\gamma_5)$ $\quad P_L = \frac{1}{2}(1-\gamma_5)$

are projection operators as

$$P_R + P_L = \frac{1}{2}\left(1+\gamma_5 + 1 - \gamma_5\right) = 1$$

$$P_{\substack{R\\L}}^2 = \frac{1}{4}\left(1 \pm 2\gamma_5 + \underbrace{\gamma_5\gamma_5}_{1}\right) = \frac{1}{2}\left(1 \pm \gamma_5\right) = P_{\substack{R\\L}}$$

$$P_R P_L = \frac{1}{4}(1+\gamma_5)(1-\gamma_5) = \frac{1}{4}(1-\gamma_5\gamma_5) = 0.$$
$$= P_L P_R$$

b) $u_\pm = \sqrt{E}\begin{pmatrix}\chi_\pm \\ \pm\chi_\pm\end{pmatrix}$ in the UR limit.

$\gamma_5 = \begin{pmatrix} 0 & \mathbb{1} \\ \mathbb{1} & 0 \end{pmatrix}$ (in the Dirac basis that is used here)

$P_R = \frac{1}{2}(1+\gamma_5) = \frac{1}{2}\begin{pmatrix} \mathbb{1} & \mathbb{1} \\ \mathbb{1} & \mathbb{1} \end{pmatrix}$, $\quad P_L = \frac{1}{2}\begin{pmatrix} \mathbb{1} & -\mathbb{1} \\ -\mathbb{1} & \mathbb{1} \end{pmatrix}$

$$P_R u_+ = \frac{1}{2}\begin{pmatrix} \mathbb{1} & \mathbb{1} \\ \mathbb{1} & \mathbb{1} \end{pmatrix}\sqrt{E}\begin{pmatrix}\chi_+ \\ \chi_+\end{pmatrix} = \sqrt{E}\begin{pmatrix}\chi_+ \\ \chi_+\end{pmatrix} = u_+$$

$$P_L u_- = \frac{1}{2}\begin{pmatrix} \mathbb{1} & -\mathbb{1} \\ -\mathbb{1} & \mathbb{1} \end{pmatrix}\sqrt{E}\begin{pmatrix}\chi_- \\ -\chi_-\end{pmatrix} = \sqrt{E}\begin{pmatrix}\chi_- \\ -\chi_-\end{pmatrix} = u_-$$

3. a) use: $\{\gamma^\mu, \gamma_5\} = 0$

$\Rightarrow \quad P_{R/L} \cdot \gamma^\mu = \frac{1}{2}(1 \pm \gamma_5)\gamma^\mu = \gamma^\mu \frac{1}{2}(1 \mp \gamma_5) = \gamma^\mu P_{L/R}$

$$\bar{q}\gamma^\mu q = \bar{q}(P_R + P_L)\gamma^\mu(P_R + P_L)q$$

$$= \bar{q}\Big(\underbrace{P_R\gamma^\mu P_R}_{\gamma^\mu P_L P_R = 0} + P_R\gamma^\mu P_L + P_L\gamma^\mu P_R + \underbrace{P_L\gamma^\mu P_L}_{\gamma^\mu P_R P_L = 0}\Big)q$$

$$= \bar{q} P_R \gamma^\mu P_L q + \bar{q} P_L \gamma^\mu P_R q$$

$$= \bar{q}_L \gamma^\mu q_L + \bar{q}_R \gamma^\mu q_R$$

b) $$\bar{q}\,\mathbb{1}\,q = \bar{q}\underbrace{(P_R + P_L)(P_R + P_L)}_{P_R P_R + P_L \cdot P_L} q = \bar{q} P_R P_R q + \bar{q} P_L P_L q$$

$$= \bar{q}_L q_R + \bar{q}_R q_L.$$

4. $$\delta \mathcal{L} = -i \Big[\bar{q}_R \left(\sum_a \theta^R_a \frac{\lambda_a}{2} + \theta^R \right) M q_L$$
$$- \bar{q}_R M \left(\sum_a \theta^L_a \frac{\lambda_a}{2} + \theta^L \right) q_L$$
$$+ \bar{q}_L \left(\sum_a \theta^L_a \frac{\lambda_a}{2} + \theta^L \right) M q_R$$
$$- \bar{q}_L M \left(\sum_a \theta^R_a \frac{\lambda_a}{2} + \theta^R \right) q_R \Big]$$

$$= -i \Big[\sum_a \theta^R_a \left(\bar{q}_R \frac{\lambda_a}{2} M q_L - \bar{q}_L M \frac{\lambda_a}{2} q_R \right)$$
$$+ \theta^R \left(\bar{q}_R M q_L - \bar{q}_L M q_R \right)$$
$$+ \sum_a \theta^L_a \left(\bar{q}_L \frac{\lambda_a}{2} M q_R - \bar{q}_R M \frac{\lambda_a}{2} q_L \right)$$
$$+ \theta^L \left(\bar{q}_L M q_R - \bar{q}_R M q_L \right) \Big]$$

$$\frac{\partial \mathcal{L}}{\partial \theta^L_a} = -i \left(\bar{q}_L \frac{\lambda_a}{2} M q_R - \bar{q}_R M \frac{\lambda_a}{2} q_L \right) \equiv \partial_\mu L^{\mu, a}$$

$$\frac{\partial \mathcal{L}}{\partial \theta^R_a} = -i \left(\bar{q}_R \frac{\lambda_a}{2} M q_L - \bar{q}_L M \frac{\lambda_a}{2} q_R \right) \equiv \partial_\mu R^{\mu, a}$$

$$\frac{\partial \mathcal{L}}{\partial \theta^L} = -i \left(\bar{q}_L M q_R - \bar{q}_R M q_L \right) \equiv \partial_\mu L^\mu$$

$$\frac{\partial \mathcal{L}}{\partial \theta^R} = -i \left(\bar{q}_R M q_L - \bar{q}_L M q_R \right) \equiv \partial_\mu R^\mu$$

$$V^{\mu,a} = R^{\mu,a} + L^{\mu,a}$$

$$\partial_\mu V^{\mu,a} = -i \Big(\bar{q}_R \frac{\lambda_a}{2} M q_L - \bar{q}_L M \frac{\lambda_a}{2} q_R$$

$$+ \bar{q}_L \frac{\lambda_a}{2} M q_R - \bar{q}_R M \frac{\lambda_a}{2} q_L \Big)$$

$$= -i \, \bar{q}_R \left[\frac{\lambda_a}{2}, M \right] q_L$$

$$-i \, \bar{q}_L \underbrace{\left[\frac{\lambda_a}{2}, M \right]}_{\sim 1, \text{ so mixes chirality states}} q_R$$

$$= -i \bar{q} \left[\frac{\lambda_a}{2}, M \right] q = \underline{i \bar{q} \left[M, \frac{\lambda_a}{2} \right] q}$$

$$A^{\mu,a} = R^{\mu,a} - L^{\mu,a}$$

$$\partial_\mu A^{\mu,a} = -i \, \bar{q}_R \left\{ \frac{\lambda_a}{2}, M \right\} q_L + i \, \bar{q}_L \left\{ \frac{\lambda_a}{2}, M \right\} q_R$$

$$= -i \, q P_L \left\{ \frac{\lambda_a}{2}, M \right\} P_L q + i q P_R \left\{ \frac{\lambda_a}{2}, M \right\} P_R q$$

$$P_L^2 = P_L \qquad P_R^2 = P_R$$

$$= -i q \left\{ \frac{\lambda_a}{2}, M \right\} \frac{1}{2} (1 - \gamma_5) q + i q \left\{ \frac{\lambda_a}{2}, M \right\} \frac{1}{2} (1 + \gamma_5) q$$

$$= -i q \left\{ \frac{\lambda_a}{2}, M \right\} \left(-\frac{1}{2} \gamma_5\right) q + i q \left\{ \frac{\lambda_a}{2}, M \right\} \frac{1}{2} \gamma_5 q$$

$$= i q \left\{ \frac{\lambda_a}{2}, M \right\} \gamma_5 q$$

$$\partial_\mu V^\mu = \partial_\mu R^\mu + \partial_\mu L^\mu$$
$$= -i\left(\bar{q}_R M q_L - \bar{q}_L M q_R + \bar{q}_L M q_R - \bar{q}_R M q_L\right)$$
$$= 0$$

and

$$\partial_\mu A^\mu = \partial_\mu R^\mu - \partial_\mu L^\mu = -i\left(2\underbrace{\bar{q}_R M q_L}_{-\frac{1}{2}\bar{q} M \gamma_5 q \text{ as before}} - 2\overbrace{\bar{q}_L M q_R}^{+\frac{1}{2}\bar{q} M \gamma_5 q}\right)$$

$$= 2i\bar{q} M \gamma_5 q .$$ ($\neq$ quantum)

(see also hep-ph/0503263 p. 23)

Thus

$\partial_\mu V^\mu = 0$: vector current is conserved exactly
$\therefore$ baryon number is conserved

$\partial_\mu A^\mu \sim M$, conserved for massless quarks
$\to$ quantum effects break

If quark masses are equal $\left[\frac{\lambda_a}{2}, M\right] \sim [\lambda_a, \mathbb{1}] = 0$,
and $\partial_\mu A^{\mu,a} = \partial_\mu V^{\mu,a} = 0$