Exercise 5 Return by 16.00 on Tue 30.10.

1. The spherical harmonic functions Y_{lm} are eigenfuctions of parity with eigenvalues $P_a(-1)^l$. Show, using the definition for Y_{lm}, that the reflection $\vec{x} \to -\vec{x}$ results in $Y_{lm}(\pi - \theta, \varphi + \pi) = (-1)^l Y_{lm}(\theta, \varphi)$.

2. Let's practise the addition of angular momenta.

   a) Using the Clebsch-Gordan table at
   `http://pdg.lbl.gov/2009/reviews/rpp2009-rev-clebsch-gordan-coefs.pdf`
   form the states $|J_1\, J_2\, J\, M\rangle_c$ in the coupled basis, when

   i) $j_1 = j_2 = \frac{1}{2}$
   ii) $j_1 = j_2 = 1$.

   b) Let's couple three angular momenta $J_1 = J_2 = J_3 = \frac{1}{2}$. Find all possible states in the coupled basis $|J_1\, J_2\, (J_{12})\, J_3\,;\, J\, M\rangle_c$, where you have first coupled J_1 and J_2. Verify the dimensional decomposition $2 \otimes 2 \otimes 2 = 4 \oplus 2 \oplus 2$ given on page 140(iii) in the lectures.

3. The $p\bar{p}$ annihilation happens at rest through the S-states. Explain using parity why the process $p\bar{p} \to \pi^0\pi^0$ cannot happen through the strong interaction.

4. The $\eta(547)$ meson is a spin-0 particle which decays through the electromagnetic (or strong) interaction to three pions:

   $$\eta \to \pi^0 + \pi^0 + \pi^0$$

   $$\eta \to \pi^+ + \pi^- + \pi^0.$$

   Figure out the parity P_η of the η and explain why the decay processes

   $$\eta \to \pi^+ + \pi^-$$

   $$\eta \to \pi^0 + \pi^0$$

   are not observed.

5. a) If a meson M decays via the strong interaction to two pions $\pi^+\pi^-$, show that then the relation $P_M = C_M = (-1)^{J_M}$ holds.

   b) The mesons $\rho^0(770)$ and $f_2^0(1275)$ decay through the strong interaction to a pion pair $\pi^+\pi^-$. The ρ has spin $J_\rho = 1$ and the f_2 has spin $J_{f_2} = 2$. Are the decays $\rho^0 \to \pi^0\gamma$ and $f_2^0 \to \pi^0\gamma$ possible through the electromagnetic interaction? Are the decays $\rho^0 \to \pi^0\pi^0$ and $f_2^0 \to \pi^0\pi^0$ possible through any interaction?