Exercise 3 Return by 16.00 on Tue 16.10.

1. a) The HERA accelerator at DESY in Germany made electron-proton collision experiments with energies $E_e = 30$ GeV and $E_p = 920$ GeV. What was $\sqrt{s}$ in these experiments? What would the corresponding projectile particle energy be in a fixed target experiment if the target was i) the proton or ii) the electron.

   b) The RHIC accelerator at BNL in the US and the LHC at CERN both collide proton beams. In RHIC the center of mass energy is $\sqrt{s} = 500$ GeV and the corresponding value for the LHC is $\sqrt{s} = 8$ TeV. How large projectile particle energies would you need if these were fixed target experiments?

2. Let's consider an elastic scattering $a + b \to a + b$.

   a) Derive the results given in the first box on page 42 in the lecture notes. Also figure out what is u in the same case.

   b) Derive the ultrarelativistic results given for s, t and u in the second box on page 42. Show also that $s + t + u \approx 0$.

3. Let's consider an elastic proton-proton scattering as shown on page 45 in the lecture notes. In the CMS frame we have $\theta_c^* = 60°$ and $|\vec{p}^{\,*}| = \sqrt{3}m_p$. Calculate the quantities θ_c^{TRF}, θ_d^{TRF}, $|\vec{p}_c^{\,TRF}|$, $|\vec{p}_d^{\,TRF}|$, E_c^{TRF} and E_d^{TRF}. Verify your results by checking that energy and momentum are conserved in the TRF frame.

4. Let's reverse the previous problem. Assume now that in the TRF frame we have $|\vec{p}_a^{\,TRF}| = \sqrt{48}m_p$ and $\cos\theta_c^{TRF} = 6/\sqrt{39}$. Now show that $\theta_c^* = 60°$ and $|\vec{p}^{\,*}| = \sqrt{3}m_p$ without using the results from the previous problem. What is $\sqrt{s}$?

5. a) Show that the phase space element $\mathrm{d}^3p/2E$ is invariant under Lorentz transformations i.e. rotations and boosts (you only need to consider boosts in the z-direction). Remember: Rotation matrices are orthogonal, i.e. $R^T R = \mathbf{1}$.

   b) On pages 75-76 in the lecture notes the flux factor in the target rest frame was shown to be $2E_a^{TRF} 2E_b^{TRF} |\vec{v}_a^{\,TRF}| = 2\sqrt{\lambda(s, m_a^2, m_b^2)}$. Now if we consider colliding beams with velocities $\vec{v}_a$ and $\vec{v}_b$ in opposite directions on the z-axis the flux factor would be $2E_a 2E_b |\vec{v}_a - \vec{v}_b|$. Show that also this can be written in the Lorentz invariant form $2\sqrt{\lambda(s, m_a^2, m_b^2)}$.

6. a) Show that the Klein-Gordon equation (KGE) $(\partial_\mu \partial^\mu + m^2)\psi(x) = 0$ has plane wave solutions $\psi_{\vec{p}}(x) = \mathcal{N} e^{-ip\cdot x}$, where $E^2 = \vec{p}^{\,2} + m^2$ and $\mathcal{N}$ is a normalisation constant.

   b) Show that the continuity equation $\partial_\mu j^\mu = 0$, where $j^\mu = i(\psi^* \partial^\mu \psi - \psi \partial^\mu \psi^*)$, follows from the KGE.

   c) Show that plane wave solutions for the KGE give $j^\mu = 2p^\mu |\mathcal{N}|^2$.