

**FYSH300 fall 2011**

Exercise 6, return by Mon Oct 24th at 9.00 to box in the lobby, discussed Mon Oct 24th, at 12.15  
in FYS5

1. Solve the differential equation

$$i \frac{da_0(t)}{dt} = H_{01} e^{-i(E_1 - E_0)t} - i \frac{\Gamma}{2} a_0(t), \quad (1)$$

on slide 21 in the lectures, with the initial condition  $a_0(0) = 0$ . Show that

$$P_0 \equiv |a_0(t \gg \Gamma^{-1})|^2 = \frac{|H_{01}|^2}{(E_1 - E_0)^2 + \frac{\Gamma^2}{4}}. \quad (2)$$

2. Examine the processes

$$K^- p \rightarrow K^- p \quad \text{and} \quad K^- p \rightarrow \bar{K}^0 n. \quad (3)$$

Draw the quark diagrams and figure out if a resonance is possible in these processes.

3. Assume that we have the mass matrix

$$H_m = \begin{pmatrix} m + \epsilon & \epsilon & \epsilon \\ \epsilon & m + \epsilon & \epsilon \\ \epsilon & \epsilon & m' + \epsilon \end{pmatrix}. \quad (4)$$

for the  $|u\bar{u}\rangle |d\bar{d}\rangle |s\bar{s}\rangle$  spin 1 (vector meson) states. Find out the 3 eigenvectors of the mass matrix, and their eigenvalues. Assuming these eigenstates are the  $\rho^0$ ,  $\omega^0$  and  $\phi^0$  vector mesons, find out the values of  $m, \epsilon, m'$ . You have to correctly identify which particle corresponds to which eigenvector! Recall that  $\rho^0$  is a member of the isospin triplet  $\rho^\pm, \rho^0$  which has strangeness zero.

4. Consider the process  $\gamma + p \rightarrow \pi^0 + p$ . If  $E_\gamma = 3K$  (3 Kelvin !),  $\sqrt{s} = m_\Delta$  (the  $\Delta$  resonance mass) and the collision is head-on (incoming  $p$  and  $\gamma$  have momenta in the opposite direction), what is the energy of the proton? What are the energies of the incoming and outgoing proton in the CMS? Find out and explain briefly what is the ‘‘GZK cutoff’’.

**Bonus points for calculating the full kinematics:** Assuming that the scattering angle is  $\theta = 0$ , calculate the energy of the outgoing proton in the original collision frame. One option could be to figure out the boost velocity between the original and the CMS frames from the initial proton energies and momenta, then apply this boost to the outgoing proton energy in the CMS. Or maybe you can find a more clever way?

5. Look at the explicit expressions for the SU(3) generators  $t_a$  from the lecture slides (e.g. slide 29 in section 4). Two of them are diagonal,  $t_3$  and  $t_8$ . The unit vectors  $\mathbf{e}_1 = (1, 0, 0)^T$ ,  $\mathbf{e}_2 = (0, 1, 0)^T$ ,  $\mathbf{e}_3 = (0, 0, 1)^T$  are eigenvectors of  $t_3$  and  $t_8$ . The eigenvalue of  $t_3$  is the 3rd component of isopin  $I_3$  and the eigenvalue of  $t_8$  is  $\sqrt{3}/2Y$ , where  $Y$  is the hypercharge.

- (a) Draw the vectors  $\mathbf{e}_i$  as points in the  $I_3, Y$ -plane. Assuming that now SU(3) is the SU(3) of quark flavor, identify these points as the  $u, d, s$  quarks.
- (b) We define the ladder operators  $t_{12\pm} = t_1 \pm it_2$ ;  $t_{45\pm} = t_4 \pm it_5$ ,  $t_{67\pm} = t_6 \pm it_7$ . Find out what is the result of when  $t_{ab\pm}$  operates on  $\mathbf{e}_i$ ; e.g.  $t_{12-}\mathbf{e}_1$ . Some of these operations change  $\mathbf{e}_i$  into  $\mathbf{e}_j$  with  $i \neq j$ . Draw these as arrows in the  $I_3, Y$ -plane figure. What do the other operations do?

6. Now let us consider the SU(3) of color. Show that the totally antisymmetric 3-quark color state is color neutral. In other words, show that operating on the tensor product state

$$|\chi\rangle = \mathbf{e}_1 \otimes \mathbf{e}_2 \otimes \mathbf{e}_3 + \mathbf{e}_2 \otimes \mathbf{e}_3 \otimes \mathbf{e}_1 + \mathbf{e}_3 \otimes \mathbf{e}_1 \otimes \mathbf{e}_2 - \mathbf{e}_1 \otimes \mathbf{e}_3 \otimes \mathbf{e}_2 - \mathbf{e}_3 \otimes \mathbf{e}_2 \otimes \mathbf{e}_1 - \mathbf{e}_2 \otimes \mathbf{e}_1 \otimes \mathbf{e}_3 \quad (5)$$

with the color charge operator

$$t_a^{\text{tot}} \equiv t_a \otimes 1 \otimes 1 + 1 \otimes t_a \otimes 1 + 1 \otimes 1 \otimes t_a \quad (6)$$

we get  $t_a^{\text{tot}}|\chi\rangle = 0$  for all  $a$ . It is enough to check a couple of generators, e.g.  $t_3, t_8, t_{45\pm} = t_4 \pm it_5$ .