FYSH300 fall 2011

Exercise 1, return by Mon Sep 19th at 9.00 to box in the lobby, discussed Mon Sep 19th, at 12.15 in FYS5

1. On this course we use the natural units $\hbar = c = k_B \equiv 1$. To get a feel of these units let's do some exercises on the subject.
   (a) What is the Z-boson mass 91.1 GeV in SI units?
   (b) A proton has a momentum of 1 GeV. What is this in SI units?
   (c) Assume that a neutron star has a nucleon density of 0.0013 GeV^3. How many nucleons are there in a cubic fermi?
   (d) Cross-sections are sometimes measured in *barns*. A barn is defined as 1 b $= 10^{-28}\mathrm{m}^2$ and a millibarn is then 1 mb $= 10^{-3}$ b. Calculate what is 400 mb in fm^2.
   (e) The core temperature of the sun is around 16 MK. Is this hot or cold compared to the temperature of 200 MeV reached in relativistic heavy-ion collisions? Calculate.

2. The LHC beam consists of approximately 3.2 $\times 10^{14}$ protons. Each has a maximum energy of 7000 GeV. Calculate how fast would a Formula One car have to drive (assume the car weighs 700 kg) to acquire the same amount of kinetic energy as in the LHC beam. Could it be done?

3. Consult the particle listings in the PDG web-pages for the following:
   (a) What is the heaviest meson found so far? What is it's quark content and mass?
   (b) What is the quark content of the Δ-baryons Δ^{++}, Δ^{+}, Δ^{0} and Δ^{-}? What are the corresponding antiparticles and what is their quark content?

4. (a) Draw a Feynman diagram of the decay $n \to p + e^- + \bar{\nu}_e$. Hint: Figure out the quark content of n and p and use the fact that this decay is caused by the charged weak current. Draw carefully all the lines and arrows and identify the elementary particles.
   (b) Draw a Feynman diagram for the decay $p \to n + e^+ + \nu_e$. This reaction cannot take place if $m_p < m_n + m_e + m_\nu$. Is this the case here?

5. The Maxwell equations are

$$\nabla \cdot \mathbf{E} = \rho \qquad \nabla \times \mathbf{B} - \partial_t \mathbf{E} = \mathbf{j} \tag{1}$$

$$\nabla \cdot \mathbf{B} = 0 \qquad \nabla \times \mathbf{E} + \partial_t \mathbf{B} = 0 \tag{2}$$

   First show that the homogenous equations (2) are automatically satisfied by $\mathbf{E} = -\partial_t \mathbf{A} - \nabla\phi$ and $\mathbf{B} = \nabla \times \mathbf{A}$ (i.e. $B^i = \epsilon^{ijk}\partial_j A^k$).

   Now let us write these equations in a manifestly covariant form. We define the four-vectors $j^\mu = (\rho, \mathbf{j})$ and $A^\mu = (\phi, \mathbf{A})$ and the tensor $F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu$.

   (a) Write out $F^{\mu\nu}$ as a 4×4 matrix in terms of $(t, \mathbf{x})$-derivatives of $\phi, \mathbf{A}$ and identify its components as the components of $\mathbf{E}, \mathbf{B}$.
   (b) Show that $\partial_\mu F^{\mu\nu} = j^\nu$ gives the inhomogenous Maxwell equations (1).

   Pay attention to the signs! Take care with covariant and contravariant indices! Remember that $\partial_\mu = \partial/\partial_{x^\mu}$ (index down in ∂_μ corresponds to derivative w.r.t. x^μ with index up).