Particle Physics, Part 2

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Recall from Wednesday

- "Diracology": calculate spin-summed and averaged squared amplitude $\overline{|\mathcal{M}|^2}$
- Sum over final state spins
- Average over initial state spins
- Leptonic tensors $L_{\mu\nu}$, write as traces, use $\sum_s u(p)\bar{u}(p) = \not{p} + m$
- Traces of gamma matrices
- QED $\to$ QCD: Invariant in local $SU(3)$ transforms $\exp(i\sum_{a=1}^{8}\alpha^a t^a)$
- $F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu + ig[A_\mu, A_\nu]$ is 3x3 matrix, $A_\mu = A_\mu^a t^a$, gluon color a
- Quark spinors also carry a 3−component color vector
- QCD Lagrangian

Quiz, p. 316

Quiz

Two-jet production in photon-proton scattering can be used to probe the quark and gluon densities. Draw some diagrams that produce two jets (remember: quarks and gluons become jets) and probe the gluon density of the proton:

$$\gamma + g \to q + \bar{q}$$

How can you produce two jets in photon + quark scattering?

Quiz, p. 316

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How can you produce two jets in photon + quark scattering?

Photon$\to q\bar{q}$ splitting, then quark and/or (lowest order: or) antiquark couples to the gluon from the proton.

A quark can absorb the photon, and emit a gluon (γ and g not indentical, no cross diagram!)

Quiz, p. 325

Quiz

Consider a process $q(i)\bar{q}(j) \to q(k)\bar{q}(l)$ via an s channel gluon. Here i, j, k, l refer to the colors. Calculate the color factor C (which multiplies the invariant amplitude $\overline{|\mathcal{M}|^2}$.

Quiz, p. 325

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Summation over i, j, k, l and gluon color indices a, b is implicit. Note that the gluon propagator forces color at both ends to be the same. We use $(t^a_{ij})^* = t^a_{ji}$, as $t^{a\dagger} = t^a$.

$$C = \frac{1}{N_c^2}(t^a_{ji}t^a_{kl})(t^b_{ji}t^b_{kl})^* = \frac{1}{N_c^2}(t^a_{ji}t^b_{ij})(t^a_{kl}t^b_{lk}) = \frac{1}{N_c^2}\mathrm{Tr}(t^a t^b)\mathrm{Tr}(t^a t^b) = \frac{1}{N_c^2}\frac{1}{2}\delta^{ab}\frac{1}{2}\delta^{ab}$$

$$= \frac{1}{N_c^2}\frac{1}{2\cdot 2}8 = \frac{2}{9}$$

Same result as in the t channel case.

Reading assignment

We will discuss these shortly....

- Why it is possible to probe proton structure (distance scale $\ll$ proton radius) in DIS?
- What is the interpretation (in the parton model) of Bjorken-x, defined as

$$x = \frac{Q^2}{2P \cdot q}$$

- If the structure functions satisfy $F_1 = \frac{1}{2x} F_2$, and $F_2 \sim$ number of quarks (Callan-Gross relation), why does it tell us that the quarks are spin-$\frac{1}{2}$ particles?

Recall from Monday

- QCD vertices: qqg, ggg and ggg
- Feynman rules, for qqg similar to $qq\gamma$ but an additional t^a_{ij}
- $a = 1, 2, 3, 4, 5, 6, 7, 8$ is the gluon color, and quarks carry a color index $i = 1, 2, 3$
- Cross section calculations in QCD: as in QED, but additional color factor (factorizes, can calculate separately)
- Coupling constants run, α_{em} large at large scales (small distances) $\alpha_s \to 0$ at large scales (asymptotic freedom)
- Determining α_s: QCD corrections to scattering processes at different Q scales
- Collider data: quarks are spin-$\frac{1}{2}$, gluons spin-1 (angular distributions), # of colors $N_c = 3$

Rich QCD phenomenology, we won't cover here in detail, read from the lecture notes!

Reading assignment

Quiz

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Why it is possible to probe proton structure (distance scale $\ll$ proton radius) in DIS?

Uncertainty principle: virtual photon has momentum $\sim q$, wavelength $\sim 1/q$. At HERA, one reaches high $Q^2 = -q^2$, corresponding to $1/Q \sim 1/(100\ \mathrm{GeV}) \sim 1/500$ fm. Compare with proton radius ~ 1 fm.

Lecture notes p. 334

Reading assignment

Quiz

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Quiz

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Fraction of the proton momentum carried by the quark participating in the scattering in DIS (in the frame where the proton has large longitudinal momentum).

Lecture notes p. 338

Reading assignment

Quiz

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If quarks are spin-$\frac{1}{2}$ fermions, the angular distribution in DIS should be the same as in $e + \mu \to e + \mu$ scattering (p. 336). The parametrized DIS cross section (p. 335) has this form, if $F_1 = \frac{1}{2x} F_2$, normalized by $\sim$ number of quarks.

Quiz, p. 360

Quiz

Check that the weak hypercharge for the lefthanded u quark indeed is $+1/3$

Quiz, p. 360

Quiz

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First calculate weak isospin, note that $u_L = \begin{pmatrix} u_L \\ 0 \end{pmatrix}$:

$$\hat{T}_3 u_L = \frac{1}{2}\begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}\begin{pmatrix} u_L \\ 0 \end{pmatrix} = \frac{1}{2}\begin{pmatrix} 1 \cdot u_L \\ 0 \end{pmatrix},$$

so the weak isospin $T_3 = 1/2$. Thus, the weak hypercharge reads

$$Y = 2(Q_{\text{em}} - T_3) = 2\left(\frac{2}{3} - \frac{1}{2}\right) = \frac{1}{3}.$$