Particle Physics, Part 2

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Recall from Wednesday

- Classical $\Rightarrow$ quantum field theory: 2nd quantizations, fields $\to$ operators
- Interaction terms in Lagrangian $\to$ basic vertices of the theory. E.g. $\lambda/4!\phi^4 = 4$ point interaction $\sim \lambda$
- Peturbation theory: expansion on couplings (e.g. λ)
- Lagrangian $\Rightarrow$ Feynman diagrams
- QED: Require invariance in *local* $U(1)$ transforms $\psi \to e^{i\alpha(x)}\psi$
- Requires new field A^μ, which is the gauge field of classical ED, now photon field
- Basic vertex: electron-electron-photon

Quiz

At the LHC, the process $\gamma\gamma \to \gamma\gamma$ (light-by-light scattering) is recently observed (CMS, `arXiv:1810.04602`, Phys. Lett. B797, 134826). Draw the most likely diagram that contributes to this process. How does the cross section scale with QED coupling constant α?

Quiz, p. 281

Quiz

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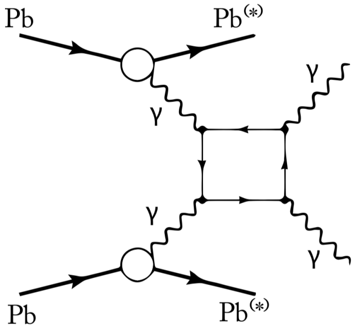

- Photon-electron vertex $\sim e$
- Graph (amplitude) $\sim e^4$
- Cross section $\sim$ graph$^2 \sim e^8 \sim \alpha^4 \sim \frac{1}{137^4} \sim 10^{-9}$.
- Note: classically forbidden!

(Nuclei emit photons, let's not consider that part here. Momentum arrows missing)

Quiz, p. 292

Quiz

Using the Feynman rules, write down the invariant amplitude $-i\mathcal{M}_{fi}$ for the process $e^+(p_a)e^-(p_b) \to \mu^+(p_c)\mu^-(p_d)$ (leading order contribution, first draw the diagram).

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$$
\begin{aligned}
&-i\mathcal{M}_{fi}(e^+(p_a)e^-(p_b) \to \mu^+(p_c)\mu^-(p_d)) \\
&= \bar{u}(p_d)(-ie\gamma_\mu)v(p_c)\frac{-ig^{\mu\nu}}{(p_a+p_b)^2}\bar{v}(p_a)(-ie\gamma_\nu)u(p_b)
\end{aligned}
$$

Note $q = p_a + p_b = p_c + p_d$

Recall from Monday

- Basic QED interactions from $ee\gamma$ vertex
- Full QED Lagrangian, with electron, muon, tau + quarks
- QED tests, excellent agreement with collider + low E (e.g. magnetic moment) tests
- Derivation of Feynman rules in a specific case (time dependent perturbation theory)
  - Start from the end of a fermion line
  - Outgoing particle: $\bar{u}(p)$
  - Incoming particle $u(p)$
  - Outgoing antiparticle $v(p)$
  - Incoming antipartilce $\bar{v}(p)$
  - Photon propagator $-ig^{\mu\nu}/q^2$
  - Vertex $-ie\gamma_\mu$.

Quiz

We just computed the leptonic tensors in the massless case. Let's prepare for the next exercises (Ex. 10, prob. 1): Calculate the contributions proportional to the muon mass m (or m^2) from the Leptonic tensor

$$L^{\mu\nu}_{\text{muon}} = \frac{1}{2}\mathrm{Tr}[(\not{p}_d + m)\gamma^\mu(\not{p}_b + m)\gamma^\nu]$$

Quiz, p. 298

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$$L^{\mu\nu}_{\text{muon}} = \frac{1}{2}\mathrm{Tr}[(\not{p}_d + m)\gamma^\mu(\not{p}_b + m)\gamma^\nu]$$

As a trace of odd number of gamma matices vanishes, $\mathrm{Tr}[\not{p}_d\gamma^\mu m\gamma^\nu] = \mathrm{Tr}\,[m\gamma^\mu\not{p}_d\gamma^\nu] = 0$. Thus we only get

$$\frac{1}{2}\mathrm{Tr}\,[m\gamma^\mu m\gamma^\nu] = \frac{m^2}{2}\mathrm{Tr}[\gamma^\mu\gamma^\nu] = 2m^2 g^{\mu\nu}.$$

Quiz, p. 312

Quiz

Verify the transformation for the gauge field components shown on p. 312. Let's only consider the term $UA_\mu U^{-1}$ (the term $\sim (\partial_\mu U)U^{-1}$ will give $\partial_\mu \alpha$ term). Using an infinitesimal transformation $U(x) = e^{i\alpha^a(x)t^a} = 1 + \alpha^a(x)t^a + \mathcal{O}(\alpha^2)$ verify (recall: $A_\mu = A_\mu^a t^a$)

$$UA_\mu U^{-1} = A_\mu^a t^a - f^{abc} t^a \alpha^b A_\mu^c$$

Quiz, p. 312

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$$UA_\mu U^{-1} = A_\mu^a t^a - f^{abc} t^a \alpha^b A_\mu^c$$

Neglecting terms $\sim \mathcal{O}(\alpha^2)$ we get

$$\begin{aligned} UA_\mu U^{-1} &= (1 + i\alpha^b t^b) A_\mu^a t^a (1 - i\alpha^c t^c) = A_\mu^a t^a + iA_\mu^a \alpha^b t^b t^a - iA_\mu^a \alpha^c t^a t^c \\ &= A_\mu^a t^a + iA_\mu^a \alpha^b t^b t^a - iA_\mu^a \alpha^b t^a t^b = A_\mu^a t^a + iA_\mu^a \alpha^b [t^b, t^a] \\ &= A_\mu^a t^a + iA_\mu^a \alpha^b i f^{bac} t^c = (A_\mu^a - f^{bca} t^a \alpha^b A_\mu^c) t^a \end{aligned}$$

Where, to get the laste quation, we exchanged $c \leftrightarrow a$ in the last term. The final result is obtained by noticing that $f^{bca} = -f^{bac} = f^{abc}$.