Particle Physics, Part 2

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Quiz

Quiz

Check your understanding, and a good place to discuss
A scientific fact: interaction enhances learning

- Quizzes are small and usually simple exercises
- Write down an answer, even if you think it is trivial
- Collaboration and discussion strongly recommended
- At the end: do you agree with your neighbour?

Recall from Wednesday

- Classical ED
  - Different gauge field configurations $A^\mu \Rightarrow$ the same physical situation $\Rightarrow$ gauge invariance
  - Charge conservation
- Basics of group theory
  - Lie groups
- $U(1)$ symmetry in case of complex scalar field $\Rightarrow$ charge conservation
- Antiparticle solutions from KGE $\Rightarrow E < 0$ particles propagating backwards in time (mathematically)

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Quiz

Check that the "negative energy plane wave" satisfies the KGE $(\Box + m^2)\phi = 0$.
Recall that the plane wave is $\phi(x) = Ne^{ip\cdot x} = Ne^{ip_\mu x^\mu}$, and now $p = (-E_p, \mathbf{p})$ with $E > 0$.

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Quiz

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Recall that the plane wave is $\phi(x) = Ne^{ip\cdot x} = Ne^{ip_\mu x^\mu}$, and now $p = (-E_p, \mathbf{p})$ with $E > 0$.

Note that $\Box = \partial_\mu \partial^\mu$. Now

$$\partial_\mu e^{ip\cdot x} = \frac{\partial}{\partial x^\mu} e^{ip_\nu x^\nu} = ip_\mu e^{ip_\nu x^\nu}$$

and

$$\partial^\mu \partial_\mu e^{ip\cdot x} = ip_\mu \underbrace{\partial^\mu}_{\frac{\partial}{\partial x_\mu}} \underbrace{e^{ip_\nu x^\nu}}_{e^{ip^\nu x_\nu}} = -p_\mu p^\mu$$

Which gives

$$(\Box + m^2)\phi = (-\underbrace{p_\mu p^\mu}_{(-E_p)^2 - \mathbf{p}^2} + m^2) Ne^{ip\cdot x} = (-m^2 + m^2) Ne^{ip\cdot x} = 0$$

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Quiz

Check the Clifford algebra $\{\gamma^\mu, \gamma^\nu\} = 2g^{\mu\nu}$ explicitly using the Dirac-Pauli representation in the case $\mu = 2, \nu = 2$. Recall that that

$$\gamma^2 = \begin{pmatrix} 0 & \sigma^2 \\ -\sigma^2 & 0 \end{pmatrix} \quad \sigma^2 = \begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix}$$

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Quiz

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$$\gamma^2 = \begin{pmatrix} 0 & \sigma^2 \\ -\sigma^2 & 0 \end{pmatrix} \quad \sigma^2 = \begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix}$$

$$\{\gamma^2, \gamma^2\} = 2\gamma^2\gamma^2 = 2\begin{pmatrix} 0 & 0 & 0 & -i \\ 0 & 0 & i & 0 \\ 0 & i & 0 & 0 \\ -i & 0 & 0 & 0 \end{pmatrix}\begin{pmatrix} 0 & 0 & 0 & -i \\ 0 & 0 & i & 0 \\ 0 & i & 0 & 0 \\ -i & 0 & 0 & 0 \end{pmatrix} = 2\begin{pmatrix} -1 & 0 & 0 & 0 \\ 0 & -1 & 0 & 0 \\ 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & -1 \end{pmatrix} = 2g^{22}\mathbb{1}$$

Or, more easily, note that $\sigma^2\sigma^2 = \mathbb{1}$ and calculate in the block form.

Quiz

Check that, in case of $i = 1$ we have $\bar{u}^{(i)}(p)u^{(i)}(p) = 2m$.
Use the 2-block form, and the fact that $(\vec{\sigma} \cdot \vec{p})(\vec{\sigma} \cdot \vec{p}) = \vec{p} \cdot \vec{p}$.

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Quiz

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Use the 2-block form, and the fact that $(\vec{\sigma} \cdot \vec{p})(\vec{\sigma} \cdot \vec{p}) = \vec{p} \cdot \vec{p}$.

Using $\chi_+^\dagger \chi_+ = 1$, $\bar{u}^1 = u^{1\dagger}\gamma^0$ we get

$$\bar{u}^1(p)u^1(p) = N(\vec{p})^2 \begin{pmatrix} \chi_+^\dagger & \frac{\vec{\sigma}\cdot\vec{p}}{E_p+m}\chi_+^\dagger \end{pmatrix} \begin{pmatrix} \mathbb{1} & 0 \\ 0 & -\mathbb{1} \end{pmatrix} \begin{pmatrix} \chi_+ \\ \frac{\vec{\sigma}\cdot\vec{p}}{E_p+m} \end{pmatrix} = N(\vec{p})^2 \begin{pmatrix} \chi_+^\dagger & \frac{\vec{\sigma}\cdot\vec{p}}{E_p+m}\chi_+^\dagger \end{pmatrix} \begin{pmatrix} \chi_+ \\ -\frac{\vec{\sigma}\cdot\vec{p}}{E_p+m} \end{pmatrix}$$

$$= N(\vec{p})^2 \left(1 - \frac{\vec{p}^2}{(E_p+m)^2}\right) = N(\vec{p})^2 \left(1 - \frac{E_p^2 - m^2}{(E_p+m)^2}\right)$$

$$= (E_p + m)\left(1 - \frac{(E_p - m)(E_p + m)}{(E_p+m)^2}\right) = E_p + m - \frac{(E_p+m)(E_p-m)}{E_p+m} = 2m$$

Recall from Monday

- Dirac equation: find Eq. linear in $\partial/\partial t$, which squared $=$ KGE
- Solution: 4-component spinors $u^{(i)}$ $\Rightarrow$ spin up+down particles + antiparticles
- γ^μ matrices and their algebra, $\{\gamma^\mu, \gamma^\nu\} = 2g^{\mu\nu}$ (anticommutator)
- Conserved current $(\partial_\mu j^\mu = 0)$ $j^\mu = \bar{\psi}\gamma^\mu\psi$
- Probability density $j^0 = \psi^\dagger\psi \;\geq 0$
- Spinor normalization and useful properties, e.g. $\sum_s u^{(s)}(p)\bar{u}^{(s)}(p) = \not{p} + m$

Quiz

Consider a real scaler field with an interaction term

$$V_I = \frac{\lambda}{4!}\phi^4 + \frac{\bar{\lambda}}{3!}\phi^3$$

Draw a few possible diagrams for $\phi\phi \to \phi\phi\phi$ scattering. How do the scattering amplitudes depend on λ and $\bar{\lambda}$? How many propagators there are in each diagram?

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Quiz

Perform a local $U(1)$ gauge transform to "photon mass term" $\frac{m_\gamma}{2} A^\mu A_\mu$ and find that it is not invariant, thus forbidden in the QED Lagrangian.
Recall Eq. (12.13): $A'_\mu = A_\mu - \frac{1}{a}\partial_\mu \alpha(x)$.

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Quiz

Perform a local $U(1)$ gauge transform to "photon mass term" $\frac{m_\gamma}{2} A^\mu A_\mu$ and find that it is not invariant, thus forbidden in the QED Lagrangian.
Recall Eq. (12.13): $A'_\mu = A_\mu - \frac{1}{a}\partial_\mu \alpha(x)$.

$$
\begin{aligned}
A^{\mu'} A'_\mu &= (A^\mu - \frac{1}{a}\partial^\mu \alpha(x))(A_\mu - \frac{1}{a}\partial_\mu \alpha(x)) \\
&= A^\mu A_\mu - \frac{1}{a}(\partial^\mu \alpha(x))A_\mu - \frac{1}{a}A^\mu(\partial_\mu \alpha(x)) + \frac{1}{a^2}(\partial_\mu \alpha(x))(\partial^\mu \alpha(x)) \\
&= A^\mu A_\mu - \frac{2}{a}(\partial^\mu \alpha(x))A_\mu + \frac{1}{a^2}(\partial_\mu \alpha(x))(\partial^\mu \alpha(x)) \neq A^\mu A_\mu
\end{aligned}
$$

As $\alpha(x)$ is arbitrary. (note: same index many times, but only twice in each product!)