

Optimal Stopping and Free-Boundary Problems

1) Contents

The lectures will disclose a fascinating connection between optimal stopping problems in probability and free-boundary problems in analysis. The focus will be on explaining the key concepts and principles through examples. The topics to be covered and linked to free-boundary problems include:

- 1) Optimal stopping problems;
- 2) Optimal prediction problems;
- 3) Nonlinear optimal stopping problems;
- 4) Quickest detection problems;
- 5) Optimal stopping games.

References: (i) Peskir, G. and Shiryaev, A. N. (2006). Optimal stopping and free-boundary problems. Lectures in Mathematics, ETH Zurich, Birkhauser, Basel (500 pp); (ii) Various research articles (2006-2023).

2) Learning outcomes

On successful completion of the course, the student will know (i) what an optimal stopping problem (OSP) is; (ii) how to classify OSPs; (iii) how to establish a one-to-one correspondence between a given OSP and a free-boundary problem (FBP); (iv) how to solve a FBP; (v) how to verify that a solution to FBP is the sought solution to OSP; (vi) how to make use of the derived solutions in various theoretical and applied settings.

3) Prerequisites

In addition to being familiar with basic probability and analysis concepts, some knowledge of (i) Markov processes (Brownian motion & Poisson process), (ii) martingales (optional sampling theorem), and (iii) stochastic calculus (Ito's formula) will be helpful although not necessary.