Stochastic Processes 2 (2008)

Examination: **7/05/2008** and **21/05/2008**
Office time for any type of questions: **28/04/2008**

(3) Martingale theory

- (3.1)
  - Lebesgue spaces $\mathcal{L}_p$
  - definition and properties of conditional expectations
  - formulation of the RADON-NIKODYM theorem
- (3.2) definition and examples of martingales
- (3.3) elementary properties of martingales
- (3.4) stopping times
- (3.5) DOOB-decomposition with proof
- (3.6) optional stopping theorem
- (3.7) DOOB's maximal inequality and KOLMOGOROV's inequality as application
- (3.8) uniformly integrable martingales
  - definition of uniformly integrable
  - Lemma 3.8.2 as sufficient condition of uniform integrability
  - characterization of closable martingales (Proposition 3.8.6) and limit theorem for sub-martingales (Corollary 3.8.9) as main theorems of this section
  - connection between almost sure convergence and L_1-convergence for u.i. sequences of random variables (Proposition 3.8.10)
- (3.9) applications of uniformly integrable martingales
  - proof of KOLMOGOROV's 0-1-law
  - branching processes
  - martingale proof of the RADON-NIKODYM theorem (the martingale property of the densities in case of refining σ-algebras based on finitely many atoms)
  - product measures
    - characterization of equivalent product measures in Proposition 3.9.7 and Corollary 3.9.8, exercise 4 from series 5
    - KAKUTANI's alternative (Proposition 3.9.10)
    - products of independent random variables as in Proposition 3.9.5 (what is the connection between products of independent random variables and densities of product measures?)
- (3.10) backward martingales
  - definition

- Example 3.10.2 for Bernoulli random variables
- convergence of backward martingales (Proposition 3.10.3)

Exercises
1: (1)
2: (1), (2), (3), (4), (5), (6), (7), (8)
3: (1), (2), (3), (4)
4: (1), (2), (3), (4), (5), (6), (7), (8)
5: (1), (2, idea), (3), (4)