

## Stochastic Differential Equations 2 (2009)

### 1. Stochastic integrals

- Definition of a stochastic integral for simple integrands and its extension to  $\mathcal{L}_2^{\text{loc}}$ .
- Properties of the stochastic integral (Proposition 3.1.25)
- Doob's maximal inequality (Proposition 3.1.16).

### 2. Itô's formula

- Definition of an Itô-process and the uniqueness of its decomposition (Proposition 3.2.7).
- Itô's formula (Proposition 3.2.9).
- Applications of Itô's formula
  - Compensator (Example 3.2.11).
  - Exponential martingales (Example 3.2.12).
  - Partial integration (Example 3.2.13).
  - Connection to PDEs (Example 3.2.14).

### 3. Stochastic differential equations

- Definition of a SDE.
- Uniqueness and existence (Propositions 4.2.2 and 4.3.1).
- Gronwall's lemma (Lemma 4.2.1).
- Criteria of Yamada and Tanaka for uniqueness (Proposition 4.2.3).
- Lévy's characterization of the Brownian motion (Proposition 4.4.5).
- Girsanov theorem (Proposition 4.4.6).
- Method of transformation of drift to solve a SDE, connection of this method to weak solutions.
- Concept of a weak solution.
- Weak solution and pathwise uniqueness imply the existence of strong solutions (Proposition 4.5.3).

**Literature:**

- [1] I. Karatzas and A. Shreve: Brownian motion and stochastic calculus (Springer)
- [2] D. Revuz and M. Yor: Continuous martingales and Brownian motion (Springer)

**Exercises:**

- (7) 4
- (8) 1, 2
- (9) 1, 2
- (10) 1, 2, 3
- (11) 1, 2, 3, 4

**Examination:** 6.5.2009

**Extra office time:** 5.5.2009; 14:00-16:00