Stokastiset differentiaaliyhtälöt

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Assume that B is a standard Brownian motion defined on a stochastic basis $(\Omega, \mathcal{F}, \mathbb{P}, (\mathcal{F}_t)_{t\geq 0})$ satisfying the usual assumptions.

(1) Consider the SDE

$$dS_t = S_t dB_t + (\mu S_t - tS_t)dt \quad \text{where} \quad S_0 = 1$$

and let $T > 0$.

(a) Solve the SDE by the help of Proposition 4.4.9 (S_t is a function, which depends on B_t, μ, and t only).

(b) Check the solution by Itô's formula.

(2) Consider the differential equation

$$dX_t = 2X_t dB_t + 5X_t dt \quad \text{with} \quad x_0 = 1.$$

(a) Find a measure Q equivalent to P such that the solution X is a martingale with respect to Q.

(b) Given an arbitrary solution X. Why does X have almost surely positive trajectories.