

Stochastic Partial Differential Equations

Exercises 11 for the 23-th of April 2002

1. Assume that the process $(\widehat{W}_{s,r})_{s,t \in \mathbb{R}}$ is defined as in the lecture and define

$$W_{st} := \widehat{W}_{s,t} - \widehat{W}_{s,0} - \widehat{W}_{0,t}$$

for $s, t \geq 0$. Show that $(W_{st})_{s,t \geq 0}$ is a 2-dimensional Brownian sheet (see Section 2.1).

2. Assume that $X = (X_t)_{t \geq 0}$ and $Y = (Y_t)_{t \geq 0}$ are processes such that all trajectories have left-hand side limits and are right-continuous. Assume that

$$\mathbb{P}(X_t = Y_t) = 1$$

for all $t \geq 0$. Show that

$$\mathbb{P}(X_t = Y_t, t \geq 0) = 1.$$