Stefan Geiß

Stochastic processes with applications to Stochastic Finance

Exercises 9 for the 19-th of November 2001

1. Let μ be a Borel-measure with an exponential distribution, that means

$$\mu(B) = c \int_B \chi_{[0,\infty)}(x) e^{-cx} dx$$

   for some $c > 0$.

   (a) One computes $\int_{\mathbb{R}} x d\mu(x)$ and $\int_{\mathbb{R}} x^2 d\mu(x)$.

   (b) One shows that $\mathbb{P}(f > s+t \,|\, f > s) = \mathbb{P}(f > t)$ for $s, t \geq 0$.

   (c) Show that the density of $\mu * \mu * \cdots * \mu$ (n times) is the density of the Gamma-distribution with parameters n and c.

2. Let $\mu(B) = e^{-c} \sum_{\{k \geq 0 : k \in B\}} \frac{c^k}{k!}$ the Poisson distribution.

   (a) Compute $\int_{\mathbb{R}} x d\mu(x) = e^{-c} \sum_{k \geq 0} k \frac{c^k}{k!}$.

   (b) Show that the Poisson distribution is infinitely divisible.