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Stochastic processes with applications to Stochastic Finance

Exercises 11 for the 3-th of December 2001

Information:

A For the final examination the course-material until the 28-th of November is relevant.

B On the 10-th of December the whole Exercise session will be used to clarify questions concerning the examination.

Problems:

1. Let N, S_1, S_2,... be independent random variables where the image distribution of N is a Poisson distribution with parameter $c > 0$ and where S_1, S_2, ... all have the same distribution such that $\mathbb{E}|S_1| < \infty$. Show that

$$\mathbb{E}\left(\prod_{i=1}^{N} S_i\right) = e^{c(\mathbb{E}S_1 - 1)}.$$

(We let $\prod_{i=1}^{0} := 1$.)

Hint: Write the expectation as a double expectation with respect to N and the S_i. Then use the definition of the Poisson distribution. Finally use the independence of the S_i to compute $\mathbb{E}\left(\prod_{i=1}^{k} S_i\right)$.

2* Let $X = (X_t)_{t\geq 0}$ be a compound Poisson process with parameters (c, ν_0) constructed like in Theorem 2.4.1 of the course. Define the new process

$$Y_t := \sum_{\substack{0\leq s\leq t,\\ 1\leq X_s - X_{s-}\leq 2}} (X_s - X_{s-}),$$

where $X_{s-}(\omega) = 0$ if $s = 0$ and $X_{s-}(\omega) = \lim_{t\uparrow s} X_t(\omega)$ if $s > 0$.

($(Y_t)_{t\geq 0}$ is the process $(X_t)_{t\geq 0}$, but one only takes jumps $X_s - X_{s-}$ if the size of the jump is in the interval $[1, 2]$.)

If μ_t is the image measure of Y_t, show that

$$\hat{\mu}(s) = e^{tc\int_1^2 [e^{isx}-1]d\nu_0(x)}.$$