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Stochastic processes with applications to Stochastic Finance

Exercises 1 for the 17-th September 2001

Definition 1 *Let* $0 < \alpha \leq 2$. *A random variable* $f : \Omega \to \mathbb{R}$ *is called* strictly α-stable *provided that*

$$\mathbb{P}(f_1 + \cdots + f_n > \lambda) = \mathbb{P}(n^{\frac{1}{\alpha}} f > \lambda)$$

for all $\lambda \in \mathbb{R}$, *where* f_1, f_2,..., f_n *are independent copies of* f.

In other words: $n^{\frac{1}{\alpha}} f$ and $f_1 + \cdots + f_n$ have the same distribution.

(1) Assume a standard Gaussian random variable $g : \Omega \to \mathbb{R}$, that means

$$\mathbb{P}(g \leq \lambda) = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{\lambda} e^{-\frac{x^2}{2}} dx.$$

Show that g is strictly 2-stable. Is g strictly α-stable for other α? (For the latter no computation is needed!)

Hint: You can use that the sum of two inpependent centered Gaussian random variables is a centered Gaussian random variable as well.

(2) Assume that $\mathbb{P}(\omega \in \Omega : -2 \leq f(\omega) \leq 2) = 1$ and $\mathbb{P}(\omega \in \Omega : 1/2 \leq f(\omega) \leq 1) > 0$. Show that f is not α-stable for $1 < \alpha \leq 2$.

Hint: Show that $f_1 + \cdots + f_n$ takes values in the interval $[n/2, n]$ with positive probability and consider then $n^{-\frac{1}{\alpha}}(f_1 + \cdots + f_n)$.