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Probability 2
Exercises 9 (9. November 2006)

Solve 4 of the 5 problems:

(1) Compute $(\chi_{[a,b]} * \chi_{[c,d]})(x)$, where $-\infty < a < b < \infty$ and $-\infty < c < d < \infty$.

(2) Compute $\mu * \nu \in \mathcal{M}_1^+(\mathbb{R})$, where

$$\mu(B) := \int_{\mathbb{R}} \chi_{[a,b]}(x)\chi_B(x)d\lambda_1(x), \quad \nu(B) := \int_{\mathbb{R}} \chi_{[c,d]}(x)\chi_B(x)d\lambda_1(x),$$

$b - a = 1$ and $d - c = 1$.

Hint: Maybe (1) and a proposition from the course.

(3) Compute $\mu * \mu * \cdots * \mu$ (n times), where $\mu := p\delta_0 + (1-p)\delta_1$, $0 < p < 1$ and $\delta_a \in \mathcal{M}_1^+(\mathbb{R})$ is the Dirac-measure in the point $a \in \mathbb{R}$.

Hint: Binomial distribution.

(4) Show that

$$(\varphi * \varphi)(x) = \frac{1}{\sqrt{4\pi}} e^{-\frac{x^2}{4}}$$

if $\varphi(x) := \frac{1}{\sqrt{2\pi}} e^{-\frac{x^2}{2}}$.

(5) Given a Borel measurable function $f : \Omega :\to C$ (that means, f considered as map between $\Omega \to \mathbb{R}^2$ is measurable with respect to $\mathcal{B}(\mathbb{R}^2)$) such that $\int_\Omega |f(\omega)| d\mathbb{P}(\omega) < \infty$. Show that

$$\left| \int_\Omega f(\omega) d\mathbb{P}(\omega) \right| \leq \int_\Omega |f(\omega)| d\mathbb{P}(\omega).$$