

Non-Life Insurance Mathematics

Exercises for

1.12.09 16:15-17:45 (MaD 380)

-5-

- (1) Assume $f \sim \text{Pois}(\lambda)$. Show that for the *moment generating function* $m_f(h)$ it holds:

$$m_f(h) = e^{-\lambda(1-e^h)}.$$

- (2) Assume the probability space $(\Omega, \mathcal{F}, \mathbb{P})$ such that $\Omega = [0, 1]$, \mathcal{F} is the Borel σ -algebra and \mathbb{P} the Lebesgue measure (especially it holds $\mathbb{P}((a, b)) = b - a$ for all $0 \leq a < b \leq 1$ and $\mathbb{P}(\{x\}) = 0$ for all $x \in [0, 1]$). Define

$$f(x) := \mathbb{1}_{[0, \frac{1}{2}]}(x), \quad g(x) := \mathbb{1}_{(\frac{1}{2}, 1]}(x)$$

and

$$h(x) := \mathbb{1}_{(0, \frac{1}{2}]}(x).$$

Which relations are true?

- (a) $f = g$ a.s.
 - (b) $\mathbb{P}(\{\omega \in \Omega : f(\omega) = h(\omega)\}) = 1$
 - (c) $f = h$ a.s.
 - (d) $f \stackrel{d}{=} 1 - g$
 - (e) $f \stackrel{d}{=} g$
- (3) (a) Determine the Regions R_1, R_2 and R_3 of points (a, b) for which the (a, b) -condition

$$q_n = \mathbb{P}(N = n) = \left(a + \frac{b}{n}\right)q_{n-1},$$

is satisfied for the Poisson, binomial and negative binomial distribution, respectively.

- (b) Show that the Poisson, binomial and negative binomial distributions are the only distributions on $\{0, 1, \dots\}$ who satisfy the '(a,b)-condition'.

Hint: Check for which points $(a, b) \notin R_1 \cup R_2 \cup R_3$ the '(a,b)-condition' defines a probability measure on $\{0, 1, \dots\}$.

- (4) Consider a reinsurance company with risk process $U(t) = u + ct - S(t)$, where $S(t)$ corresponds to an *excess-of-loss* treaty, i.e.

$$S(t) = \sum_{k=1}^{N(t)} (X_k - x)^+.$$

Assume that N is a Poisson process with parameter λ , independent of the iid sequence (X_k) , for which it holds $X_1 \sim \text{Exp}(\gamma)$. If the premium rate c is chosen according to the *expected value principle* then

$$c = (1 + \rho)\lambda\mathbb{E}(X_1 - x)^+, \quad \rho > 0.$$

- (a) Show that $c = (1 + \rho)\lambda e^{-\gamma x} \gamma^{-1}$.
(b) Show that the characteristic function of $(X_1 - x)^+$ is

$$\varphi_{(X_1 - x)^+}(t) := \mathbb{E}e^{it(X_1 - x)^+} = 1 + \frac{it}{\gamma - it}e^{-x\gamma}, \quad t \in \mathbb{R}.$$