Non-Life Insurance Mathematics

Exercises for
4.12.07 **16:00-17:30** (MaD 380)
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(1) Assume $f \sim \text{Pois}(\lambda)$. Show that for the *moment generating function* $m_f(h)$ it holds:

$$m_f(h) = e^{-\lambda(1-e^h)}.$$

(2) Assume the probability space $(\Omega, \mathcal{F}, \mathbb{P})$ such that $\Omega = [0,1]$, $\mathcal{F}$ is the Borel $\sigma-$algebra and $\mathbb{P}$ the Lebesgue measure (especially it holds $\mathbb{P}((a,b)) = b-a$ for all $0 \leq a < b \leq 1$ and $\mathbb{P}(\{x\}) = 0$ for all $x \in [0,1]$). Define

$$f(x) := \mathbb{1}_{[0,\frac{1}{2}]}(x), \qquad g(x) := \mathbb{1}_{(\frac{1}{2},1]}(x)$$

and

$$h(x) := \mathbb{1}_{(0,\frac{1}{2}]}(x).$$

Which relations are true?

(a) $f = g$ a.s.
(b) $\mathbb{P}(\{\omega \in \Omega : f(\omega) = h(\omega)\}) = 1$
(c) $f = h$ a.s.
(d) $f \stackrel{d}{=} 1 - g$
(e) $f \stackrel{d}{=} g$

(3) (a) Determine the Regions R_1, R_2 and R_3 of points (a,b) for which the (a,b)-condition

$$q_n = \mathbb{P}(N = n) = \big(a + \frac{b}{n}\big) q_{n-1},$$

is satisfied for the Poisson, binomial and negative binomial distribution, respectively.

(b) Show that the Poisson, binomial and negative binomial distributions are the only distributions on $\{0, 1, \ldots\}$ who satisfy the '(a,b)-condition'.

Hint: Check for which points $(a, b) \notin R_1 \cup R_2 \cup R_3$ the '(a,b)-condition' defines a probability measure on $\{0, 1, \ldots\}$.

(4) Consider a reinsurance company with risk process $U(t) = u + ct - S(t)$, where $S(t)$ corresponds to an *excess-of-loss* treaty, i.e.

$$S(t) = \sum_{k=1}^{N(t)} (X_k - x)^+.$$

Assume that N is a Poisson process with parameter λ, independent of the iid sequence (X_k), for which it holds $X_1 \sim \mathrm{Exp}(\gamma)$. If the premium rate c is chosen according to the *expected value principle* then

$$c = (1+\rho)\lambda \mathbb{E}(X_1 - x)^+, \qquad \rho > 0.$$

(a) Show that $c = (1+\rho)\lambda e^{-\gamma x}\gamma^{-1}$.

(b) Show that the characteristic function of $(X_1 - x)^+$ is

$$\varphi_{(X_1-x)^+}(t) := \mathbb{E}e^{it(X_1-x)^+} = 1 + \frac{it}{\gamma - it} e^{-x\gamma}, \quad t \in \mathbb{R}.$$