Non-Life Insurance Mathematics

Exercises for
05.12.06 16:15-18:00 (MaD 380)
-3-

(1) Assume $f \sim \text{Pois}(\lambda)$. Show that for the *moment generating function* $m_f(h)$ it holds:

$$m_f(h) \quad = \quad e^{-\lambda(1-e^h)}.$$

Hint: Start

$$m_f(h) \quad := \quad \mathbb{E}e^{hf} = e^{-\lambda}\sum_{k=0}^{\infty} e^{hk}\frac{\lambda^k}{k!}$$

(2) How to get a Poisson distribution from a uniform distribution? Think you have a computer which does not have the Poisson distribution but only gives you a "uniform distributed" value r on the interval $[0, 1]$, i.e.

$$\mathbb{P}(a < r \leq b) = b - a \quad \text{for } 0 \leq a < b \leq 1.$$

Consider the following procedure:
Define for $i = 1, 2, ...$ the values $a_i := e^{-\lambda}\sum_{k=0}^{i-1}\frac{\lambda^k}{k!}$ and $a_0 := 0$. Let the computer return the number n in case $a_n < r \leq a_{n+1}$ for $n = 0, 1, ...$ Show that

$$n \sim \text{Pois}(\lambda).$$

(3) Show that the Pareto distribution, given by its distribution function,

$$F(x) = 1 - \left(\frac{b}{x}\right)^a, \quad a > 0, b > 0, x \geq b,$$

is 'heavy tailed'.

Hint: Use the definition of 'heavy tailed' in chapter 5.

(4) Assume f_1, f_2, and f_3 are random variables. Decide whether the following assertions are true or false (why?)

   (a) If f_1, f_2, f_3 are independent, then f_i is independent from f_k for all $k \neq i$.

   (b) If $f_1, f_2, f_3 : \Omega \to \{0, 1, ...\}$ are independent, then f_1 is independent from $f_2 + f_3$.
   (Check whether $\mathbb{P}(f_1 = k, f_2+f_3 = l) = \mathbb{P}(f_1 = k)\mathbb{P}(f_2+f_3 = l)$)

   (c) If

$$\left.\begin{array}{l} f_1 \text{ is independent from } f_2, \\ f_1 \text{ is independent from } f_3, \\ f_2 \text{ is independent from } f_3 \end{array}\right\} \text{ then } f_1, f_2, f_3 \text{ are independent.}$$

(5) (a) What is the Cramér Lundberg model (=CLM)?

   (b) Assume an insurance company follows the 'expected value principle' with a safety loading ρ. Use Proposition 3.3.1. to show that in case of the Cramér Lundberg model it holds for the premium rate c

$$c = (1+\rho)\frac{\mathbb{E}X_1}{\mathbb{E}W_1}.$$