Models in Financial Mathematics

Exercises for
28.10.08 16:05-17:35 (MaD 380)
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`Finance`

(1) Assume the CRR-model with $S_0 = 1$, interest rate $r = 0.1$ and

$$S_{t+1} = \begin{cases} 2S_t & \text{with probability } \frac{1}{3} \\ \frac{1}{2}S_t & \text{with probability } \frac{2}{3}. \end{cases}$$

(a) Compute the EMM (equivalent martingale measure) $\mathbb{E}_{\mathbb{Q}}$.

(b) Draw the "tree" for the values of (S_t) for $t = 0, 1, 2, 3$ and write the according probabilities for the values of S_3 using $\mathbb{E}_{\mathbb{Q}}$ from a).

(c) Compute the arbitrage price of the European call-option $H = (S_3 - K)^+$ for a strike price $K = 2$.
Hint: Use the CRR pricing formula from the course (section 3.8).

(2) Assume the CRR-model, where the EMM $\mathbb{Q}$ is given by

$$\mathbb{Q}\left(\frac{S_t}{S_{t-1}} = 1 + b\right) = p.$$

The binary option (or digital option)

$$H = c\mathbb{1}_{[K,\infty)}(S_T)$$

(where $c > 0$ and $K > 0$) is sometimes also called the "cash-or-nothing option". The option holder gets c (cash) if $S_T \geq K$. On the other hand, if $S_T < K$, he gets nothing. Show that the pricing formula for this option is

$$V_0 = \frac{c}{(1+r)^T} \sum_{k=B}^{T} \binom{T}{k} p^k (1-p)^{T-k},$$

where $B = \inf\{u \in \mathbb{N} : S_0(1+b)^u(1+a)^{T-u} \geq K\}$.
Use the formula

$$V_0 = \mathbb{E}_{\mathbb{Q}} \frac{H}{S_T^0}$$

and proceed like in the course ("Computing the pricing formula").

(3) Let A and B be independent and $\mathcal{G} := \sigma\{\mathbb{1}_B\}$. What are the elements of $\mathcal{G}$? Compute

$$\mathbb{E}[\mathbb{1}_A|\mathcal{G}]$$

(without using Thm 2 of section 3.5).

(4) Let $\mathcal{F} = \sigma\{A_1, \ldots, A_N\}$ where $A_1, \ldots, A_N$ is a partition of Ω. Assume that f_1 and f_2 are $\mathcal{F}$-measurable. Is it true that

$$\{\omega \in \Omega : f_1(\omega) = f_2(\omega)\} \in \mathcal{F}?$$

Hint: You can use Proposition 1 of section 2.5. (which came after section 3.2.)