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Stochastic Modeling

Exercises 24/02/2003

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(1) (a) Give the transition matrix T of the Markov chain corresponding to the example "the random walker" of the lecture.

(b) Compute T^2.

(c) Compute (by induction)

$$\mathbb{P}(f_n = i) = \qquad , \qquad i \in \{1,2,3,4\}$$

using a Chapman-Kolmogorov equation.

(d) Compute the probability that the walker goes one circle clockwise:

$$\mathbb{P}(f_{k+4} = 1, f_{k+3} = 4, f_{k+2} = 3, f_{k+1} = 2 | f_k = 1)$$

using the step-by-step formula.

(e) Suppose we modify "the random walker" example as follows. Assume the random walker uses **2** fair coins. Each time he tosses the first one to decide whether to go or to stay. If the first one and shows "heads", he stays where he is, if it comes up "tails", he flips the second coin to decide whether he should go clockwise or counterclockwise. What is the transition matrix and the graph for this new Markov chain?

(2) Let $(f_i)_{i=0}^{\infty}$ be a Markov chain.

(a) Show that

$$\mathbb{P}(f_{i+2} = x_{i+2}, f_{i+1} = x_{i+1} | f_i = x_i) = \\ \mathbb{P}(f_{i+2} = x_{i+2} | f_{i+1} = x_{i+1})\mathbb{P}(f_{i+1} = x_{i+1} | f_i = x_i).$$

(b) Show that

$$\mathbb{P}(f_{i+1} = x_{i+1}, f_{i-1} = x_{i-1} | f_i = x_i) = \\ \mathbb{P}(f_{i+1} = x_{i+1} | f_i = x_i)\mathbb{P}(f_{i-1} = x_{i-1} | f_i = x_i).$$

Hint for (a) and (b): Use first the replacement $\mathbb{P}(A|B) = \frac{\mathbb{P}(A \cap B)}{\mathbb{P}(B)}$. Then one can transform the equalities into known statements for Markov chains.

(3) Let f_0, f_1, f_2 be independent random variables with values in $X = \{1, 2, \ldots, M\}$. Then one can regard (f_0, f_1, f_2) also as a Markov chain with state space X. Suppose the (marginal) distributions are

$$p_0(l) = \mathbb{P}(f_0 = l),$$

$$p^{(1)}(l) = \mathbb{P}(f_1 = l),$$
$$p^{(2)}(l) = \mathbb{P}(f_2 = l),$$

for l=1,...,M.
Evaluate the transition matrices T_1 and T_2.
If a *homogeneous* Markov chain consists of independent random variables, what can one say about the distribution of these random variables?