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Stochastic Modeling

Exercises 08/04/2002

-9-

(1) Let $(f_i)_{i=0}^\infty$ be a homogeneous irreducible Markov chain starting in $k \in X$ and let k be positive persistent. Let

$$\begin{aligned} T_0 &:= 0, \\ T_1 &:= \inf\{i \geq 1 | f_i = k\}, \\ T_2 &:= \inf\{i > T_1 | f_i = k\}, \\ T_3 &:= ... \end{aligned}$$

(a) Show that $T_2 - T_1$ and T_1 are independent.

(b) Show that $(T_i - T_{i-1})$, $i = 1, 2, ..$ are independent.

(c) Show using the *strong law of large numbers*:

$$\lim_{m\to\infty} \frac{T_m}{m} = \mathbb{E}T_1, \quad a.s.$$

(d) Proof of Proposition 2.6.9. under the condition $\mathbb{P}(f_0 = k) = 1$.

(2) Assume a homogeneous Markov chain $(f_i)_{i=0}^\infty$ with state space $X = \{0, ..., N\}$ and transition matrix

$$T = \begin{pmatrix} \frac{1}{N+1} & \frac{1}{N+1} & \cdots & \frac{1}{N+1} \\ \frac{1}{N+1} & \frac{1}{N+1} & \cdots & \frac{1}{N+1} \\ \cdots & \cdots & \cdots & \cdots \\ \frac{1}{N+1} & \frac{1}{N+1} & \cdots & \frac{1}{N+1} \end{pmatrix}$$

Show that each state $k \in X$ is persistent.

(3) Given a Markov chain $(f_i)_{i=0}^\infty$ with state space $X = \{0, ..., K\}$ one observes that $((f_i, i))_{i=0}^\infty$ is a homogeneous Markov chain.