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Stochastic Modeling

Exercises 04/03/2002

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(1) Let $(f_i)_{i=0}^\infty$ be a homogeneous Markov chain with state space $X = \{0, 1\}$ and transition matrix

$$T = \begin{pmatrix} \frac{1}{2} & \frac{1}{2} \\ \varepsilon & 1-\varepsilon \end{pmatrix}$$

where $0 < \varepsilon < 1$. Is the Markov chain ergodic? What are the stationary distributions?

(2) Let $(\mathcal{F}_n)_{n=0}^\infty$ be a filtration (increasing sequence of σ–algebras) and let $S, T : \Omega \to \{0, 1, 2, ...\}$ be stopping times. One shows that

(a) $\max(S, T)$ and $S + T$ are stopping times.

(b) $\mathcal{F}_T := \{A \subseteq \Omega : A \cap \{T = n\} \in \mathcal{F}_n, n = 0, 1, 2, ...\}$ is a σ–algebra.

(3) Let $(f_i)_{i=0}^\infty$ be a homogeneous Markov chain with state space $X = \{0, 1, 2\}$ and transition matrix

$$T = \begin{pmatrix} \frac{1}{2} & \frac{1}{2} & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}.$$

(a) One computes T^n for $n = 1, 2, ...$ Does T^n converge as $n \to \infty$?

(b) One computes the stationary distributions of $(f_i)_{i=0}^\infty$.