

Stokastiset mallit
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- (1) The Lebesgue measure l can be defined on intervals of the form $(a, b]$, $-\infty < a < b < \infty$ by

$$l((a, b]) := b - a.$$

Use the fact that l is a measure (continuity of a measure from below and above) to compute

- (a) $l([a, b])$
 - (b) $l((a, b))$
 - (c) $l(a, b)$
 - (d) $l(\{a \in \mathbb{R}, \text{ a rational number}\})$
- (2) Assume a homogeneous Markov chain $(f_i)_{i=1}^2$ with state space $X := \{0, 1, 2, 3\}$, transition matrix

$$T := \begin{pmatrix} 0 & 0 & \frac{1}{2} & \frac{1}{2} \\ 0 & 0 & 0 & 1 \\ \frac{1}{2} & \frac{1}{2} & 0 & 0 \\ 0 & 1 & 0 & 0 \end{pmatrix},$$

and initial distribution $(\frac{1}{2}, \frac{1}{2}, 0, 0)$. Consider the function

$$\varphi(x) := \begin{cases} x & : x \neq 2, 3 \\ 10 & : x = 2, 3 \end{cases}.$$

Is $(g_i)_{i=1}^2$ with $g_i(\omega) := \varphi(f_i(\omega))$ a Markov chain?

- (3) Olkoon $(f_i)_{i=0}^n$ Markovin ketju ja

$$0 \leq i_0 < i_1 < \dots < i_k \leq n.$$

jonon $i = 1, \dots, n$ osajono. Onko $(g_l)_{l=0}^k$ myös Markovin ketju, missä $g_l := f_{i_l}$?

- (4) Assume a branching process modelling a cell culture where the probability that the cell is dying is $p_0 \in (0, 1)$. The probability that the cell doubles is $1 - p_0$. Compute in terms of p_0

$$\mathbb{P}(f_{k+1} = i | f_k = 2), \quad \text{for } i = 0, 1, 2, 3, 4.$$

Hint: Use the probability generating function.