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Stochastic Modeling

Exercises 25/02/2002

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(1) The Lebesgue measure l can be defined on intervalls of the form $(a,b]$, $-\infty < a < b < \infty$ by

$$l((a,b]) := b - a.$$

Use the fact that l is a measure (continuity of a measure from below and above) to compute

(a) $l([a,b])$

(b) $l((a,b))$

(c) $l(a,b)$

(d) $l(\{a \in \mathbb{R}, \text{a rational number}\})$

(2) Assume a homogeneous Markov chain $(f_i)_{i=1}^2$ with state space $X := \{0,1,2,3\}$, transition matrix

$$T := \begin{pmatrix} 0 & 0 & \frac{1}{2} & \frac{1}{2} \\ 0 & 0 & 0 & 1 \\ \frac{1}{2} & \frac{1}{2} & 0 & 0 \\ 0 & 1 & 0 & 0 \end{pmatrix},$$

and initial distribution $\left(\frac{1}{2}, \frac{1}{2}, 0, 0\right)$. Consider the function

$$\varphi(x) := \begin{cases} x & : \quad x \neq 2,3 \\ 10 & : \quad x = 2,3 \end{cases}.$$

Is $(g_i)_{i=1}^2$ with $g_i(\omega) := \varphi(f_i(\omega))$ a Markov chain?

(3) Given a Markov chain $(f_i)_{i=0}^n$ and a subsequence

$$0 \leq i_0 < i_1 < \cdots < i_k \leq n.$$

Is $(g_l)_{l=0}^k$ also a Markov chain, where $g_l := f_{i_l}$?

(4) Assume a branching process modelling a cell culture where the probability that the cell is dying is $p_0 \in (0,1)$. The probability that the cell doubles is $1 - p_0$. Compute in terms of p_0

$$\mathbb{P}(f_{k+1} = i | f_k = 2), \quad \text{for} \quad i = 0,1,2,3,4.$$

Hint: Use the probability generating function.