Christel Geiss (geiss@maths.jyu.fi)

Stochastic Modeling

Exercises 18/02/2002

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(1) One gives a Maximum Likelihood Estimate for the distribution of the dry and wet days at Snoqualmie Falls for the 1–rst of January between 1948 and 1983
(the needed dates one gets from the picture about the precipitation, i.e. how many years it was dry/wet at the 1–rst of January).

(2) Given a homogeneous Markov chain $(f_i)_{i=0}^{\infty}$ with state space $X = \{0, 1\}$ and transition matrix

$$T = \begin{pmatrix} p_{00} & p_{01} \\ p_{10} & p_{11} \end{pmatrix}.$$

(a) Assume that the Markov chain is at state 0. Show that the probability to stay in this state exactly k steps (including the present step) is equal to

$$p_{00}^{k-1}(1 - p_{00}).$$

(b) One deduces that the expected number of steps to stay in this state is equal to

$$\frac{1}{1 - p_{00}}.$$

Hint: The expected number of steps to stay in state 0 equals

$$\sum_{k=1}^{\infty} k \, \mathbb{P}\,(\text{chain stays } k \text{ steps in state } 0)\,.$$

(c) What is the expected length of sequences of dry or wet days in January at Snoqualmie Falls?

(3) Assume at day 0 one person gets knowledge of a secret. Then the following happens: Since this person does not know about the importance of the secret he forgets it with probability $p_0 \in (0, 1)$ till the next day. In case the person did not forget this secret, he tells it with probability $\frac{1}{2}$ to exactly one person the next day (i.e. day 1).
Now each following day every person has forgotten the secret with probability $p_0 \in (0, 1)$ or tells the secret (under the condition he did not forget it) with probability $\frac{1}{2}$ to exactly one other person.

(a) Model the number f_i of persons knowing at day $i = 0, 1, 2, ...$ the secret as a branching process $(f_i)_{i=0}^{\infty}$ with state space $X = \{0, 1, 2, ...\}$.

(b) For what p_0 the secret will be forgotten eventually with probability $q = 1$ and $q < 1$ (compute the q) ?