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Stochastic Modeling

Exercises 11/02/2002
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(1) Let

$$p(k,l)_{k,l=0,1,2} := \begin{pmatrix} \frac{1}{3} & \frac{2}{3} & 0 \\ \frac{1}{2} & 0 & \frac{1}{2} \\ 0 & 1 & 0 \end{pmatrix}$$

be a transition matrix.

(a) Evaluate $\mathbb{P}(f_{k+3} = 1, f_{k+2} = 0, f_{k+1} = 1 | f_k = 2)$ using the step-by-step formula.

(b) Evaluate $\mathbb{P}(f_{k+4} = 1 | f_k = 1)$ using, for example, a Chapman-Kolmogorov-equation.

(2) Let $(f_i)_{i=0}^{\infty}$ be a Markov chain.

(a) Show that

$$\mathbb{P}(f_{i+2} = x_{i+2}, f_{i+1} = x_{i+1} | f_i = x_i) = \mathbb{P}(f_{i+2} = x_{i+2} | f_{i+1} = x_{i+1}) \mathbb{P}(f_{i+1} = x_{i+1} | f_i = x_i).$$

(b) Show that

$$\mathbb{P}(f_{i+1} = x_{i+1}, f_{i-1} = x_{i-1} | f_i = x_i) = \mathbb{P}(f_{i+1} = x_{i+1} | f_i = x_i) \mathbb{P}(f_{i-1} = x_{i-1} | f_i = x_i).$$

Hint for (a) and (b): Use first the replacement $\mathbb{P}(A|B) = \frac{\mathbb{P}(A \cap B)}{\mathbb{P}(B)}$. Then one can transform the equalities into known statements for Markov chains.

(3) Let f_0, f_1, f_2 be independent random variables with values in $X = \{1, 2, \ldots, M\}$. Then one can regard (f_0, f_1, f_2) also as a Markov chain with state space X. Suppose the (marginal) distributions are

$$p_0(l) = \mathbb{P}(f_0 = l),$$
$$p^{(1)}(l) = \mathbb{P}(f_1 = l),$$
$$p^{(2)}(l) = \mathbb{P}(f_2 = l),$$

for l=1,...,M.
Evaluate the transition matrices T_1 and T_2.
If a *homogeneous* Markov chain consists of independent random variables, what can one say about the distribution of these random variables?